West Essex Regional School District

Algebra 2 Honors

Summer Assignment 2023

Name ______________________________

Date _____________________

Period_________

Name ______________________________________ **Period** ____________

West Essex Regional School District

Algebra 2 Honors

Summer Assignment 2022

The topics for Algebra 2 Honors include:

- General Functions and Transformations
- Quadratic Functions
- Polynomial Functions
- Powers, Roots, and Radicals
- Rational Functions
- Exponential and Logarithmic Functions
- Conic Sections
- Trigonometry
- Sequences and Series
- Counting Principles and Probability

Algebra 2 builds on topics studied from both Algebra 1 and Geometry. There are several topics and skills that are expected to be MASTERED in any Algebra 1course and you are required to understand prior to the start of Algebra 2. This packet covers these topics, which are:

- Objective 1: Solving equations in one variable
- Objective 2: Solving and graphing inequalities in one variable
- Objective 3: Finding the slope of a line and the rate of change
- Objective 4: Graphing linear equations in slope-intercept or standard form
- Objective 5: Writing linear equations
- Objective 6: Solving linear inequalities in two variables
- Objective 7: Solving systems of linear equations
    - Graphing method
    - Substitution method
    - Elimination method
- Objective 8: Factoring completely
- Objective 9: Solving quadratic equations

Assignments

- All assignments will be collected on **The First Day Of School**, and counted toward homework grades. No credit will be given for assignments turned in late.
- There will be assessments based on this summer assignment. The dates of the assessments will be announced prior to the quizzes.

Complete all work on the assignment sheets **showing all work** for each problem and using **Pencil**, credit will not be given otherwise. It is also recommended that students use a three-ring binder for all notes, assignments, and worksheets during the year, therefore this should be the first assignment placed into the binder. Complete all problems in each objective's assignment. Although you will have some opportunity to get some help when school begins, if you find that you need a lot of help you should consider getting it before school begins. A list of helpful websites is included below.

Show all work!

I. Suggested Due Date: Week of 7/31/23

Objectives 1-2: Read pages 3-5 in the study guide.
Complete Assignment 1.

II. Suggested Due Date: Week of 8/7/23

Objectives 3-5: Read pages 6-13 in the study guide.
Complete Assignment 2.

III. Suggested Due Date: Week of 8/14/23

Objectives 6-7: Read pages 14-15 in the study guide.
Complete Assignment 3.
Read pages 16-19 in the study guide.
Complete Assignment 4.

IV. Suggested Due Date: Week of 8/21/23

Objective 8: Read pages 20-24 in the study guide.
Complete Assignment 5.

V. Suggested Due Date: Week of 8/28/23

Objective 9: Read pages 25-26 in the study guide.
Complete Assignment 6.

Here are some websites you might find useful in completing your summer assignment.

1. http://www.purplemath.com/modules/index.htm - an excellent resource for topics from pre-algebra to advanced algebra
2. http://www.khanacademy.org/ - an expansive collection of video lessons – search for any topic!
3. http://www.math.com/homeworkhelp/Algebra.html - for help with solving equations, linear equations, functions, exponents, and more
4. http://www.teacherschoice.com.au/mathematics_how-to_library.htm - for help with factoring, the quadratic formula, systems of equations, functions, and solving equations
5. http://www.algebrahelp.com/ - for help with basic algebra skills including linear equations, operations on polynomials, and factoring
6. http://www.freemathhelp.com/algebra-help.html - includes text and video lessons includes text and video lessons from an array of algebra topics including equations, inequalities, matrices, and polynomials.
7. http://mathforum.org/library/drmath/drmath.high.html - An achieve of questions and answers by Dr. Math

Algebra 2 Summer Assignment Study Guide

Objective 1: <u>Solve linear equations in one variable.</u>

Vocabulary

An **equation** is a statement that two expressions are equal.

A **linear equation** in one variable is an equation that can be written in the form $ax + b = 0$ where "a" and "b" are constants and $a \neq 0$.

A number is a **solution** of an equation if substituting the number for the variable results in a true statement.

Two equations are **equivalent equations** if they have the same solution(s).

EXAMPLE 1

Solve an equation with a variable on one side

Solve $6x - 8 = 10$

Solution

$6x - 8 = 10$	Write original equation
$6x = 18$	Add 8 to each side
$x = 3$	Divide each side by 6.

EXAMPLE 2

Solve an equation with a variable on both sides

Solve $7z + 8 = -7 - 2z - 3$

Solution

$7z + 8 = -7 - 2z - 3$	Write original equation
$7z + 8 = -2z - 10$	Simplify
$9z + 8 = -10$	Add 2z to each side.
$9z = -18$	Subtract 8 from each side
$z = -2$	Divide each side by 9.

EXAMPLE 3

Solve an equation using the distributive property

Solve $2(3x + 1) = -3(x - 2)$.

Solution

$2(3x + 1) = -3(x - 2)$	Write original equation
$6x + 2 = -3x + 6$	Distributive property
$9x + 2 = 6$	Add $3x$ to each side
$9x = 4$	Subtract 2 from each side
$x = \frac{4}{9}$	Divide each side by 9

EXAMPLE 4

Solve multiple step equations

Solve $\frac{7}{8}p - 4 = 10$ **** CLEAR FRACTIONS FIRST****

Solution

$\frac{7}{8}p - 4 = 10$	Write original equation
$8\left(\frac{7}{8}p - 4 = 10\right)$	Multiply every term by the LCD
$7p - 32 = 80$	Add 32 to each side
$7p = 112$	Divide each side by 7
$p = 16$	

EXAMPLE 5

Solve multiple step equations

Solve 3(1 – n) + 5n = 2(n + 1)

Solution

3(1 – n) + 5n = 2(n + 1)	Write original equation
3 – 3n + 5n = 2n + 2	Distributive property
3 + 2n = 2n + 2	Simplify
$3 \neq 2$	Subtract 2n from both sides
$3 \neq 2$	Correct steps may result in a false statement
∅	Conclude that the equation has no solution

Objective 2: Solve and graph linear inequalities in one variable.

Vocabulary

An **inequality** is a statement that an expression is greater (>), less than (<), greater than or equal to (≥), or less than or equal to (≤) another expression.

A **linear inequality** in one variable is an inequality that can be written in the form $ax + b > 0$, $ax + b < 0$, $ax + b \geq 0$, or $ax + b \leq 0$ where "a" and "b" are constants and $a \neq 0$.

To solve a linear inequality, follow all of the same properties of equality that you would when solving a linear equation EXCEPT:

**When you multiply or divide by a negative number, the inequality sign reverses!

Ex. 4 > 6

a. 6 > 8 add 2 to both sides, still true
b. 2 > 4 subtract 2 to both sides, still true
c. 8 > 12 multiply both sides by 2, still true
d. 2 > 3 divide both sides by 2, still true
e. -8 > -12 multiply both sides by -2, NOT true
*remember with negative numbers, the 'larger' the number, the smaller it is on the number line

A number is a **solution** of an inequality if substituting the number for the variable results in a true statement.

Since the solution of an inequality is not just one number, you can use a **graph** to indicate all of the solutions. The solutions to a linear inequality in one variable can be graphed on a number line. A dot is used to indicate the number at which the solution set begins (or ends). A closed dot indicates that the number is included in the solution. An open dot indicated that the number is not included in the solution. An arrow is drawn to the left or right of the dot indicating the numbers either less than or greater than the number.

	dot	
	closed	open
$\leq$	x	
$\geq$	x	
$<$		x
$>$		x

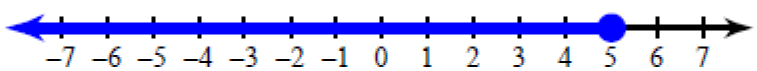
EXAMPLE 1

Solve and graph a linear inequality

Solve -2x – 2 ≥ -12

Solution

-2x – 2 ≥ -12	Write original equation
-2x ≥ -10	Add 2 to both sides
x ≤ 5	Divide each side by -2 *Remember, since you divide by a negative, the inequality sign reverses!



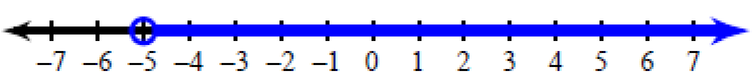
EXAMPLE 2

Solve and graph a linear inequality

Solve x + 23 > 2 – 2(x – 3)

Solution

x + 23 > 2 – 2(x – 3)	Write original equation
x + 23 > 2 – 2x + 6	Distributive property
x + 23 > 8 – 2x	Simplify
3*x* + 23 > 8	Add 2*x* to each side
3x > -15	Subtract 23 from each side
x > -5	Divide each side by 3



**Now complete Assignment 1.

Study Guide

Objective 3: <u>Find the slope of a line and the rate of change.</u>

Vocabulary

The slope *m* of a non-vertical line through points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the rise) to horizontal change (the run):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two lines are **parallel** if and only if they have the same slope, $m_1 = m_2$.

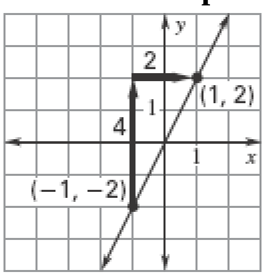
Two lines are **perpendicular** if and only if their slopes are negative (or opposite) reciprocals of each other: $m_1 = -\frac{1}{m_2}$

Slope can be used to represent an average **rate of change**, or how much one quantity changes, on average, relative to the change in another quantity, usually time.

EXAMPLE 1

Find the slope of a line

What is the slope of the line passing through the points (–1, –2) and (1, 2)?



Let $(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (1, 2)$.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-2)}{1-(-1)} = \frac{4}{2} = 2$$

The slope of the line is 2.

Practice Exercises for Example 1

Try These:

1. What is the slope of the line passing through the points (0, 0) and (1, 3)?

Solution

m = $\frac{3-0}{1-0} = 3$

2. What is the slope of the line passing through the points (2, 1) and (3, - 1)?

Solution

m = $\frac{-1-1}{3-2} = \frac{-2}{1} = -2$

EXAMPLE 2

Classify lines using slope

Without graphing, tell whether the line through the given points *rises, falls, is horizontal, or is vertical*.

a. (4, 2), (3, 2)

m = $\frac{0}{-1}$ **= 0**

Because *m* = 0, the line is horizontal.

b. (1,4), (5, 7)

m = $\frac{3}{4}$

Because *m* > 0, the line rises.

c. (4, 1), (4,-2)

m = $\frac{-3}{0}$ **= undefined**

Because *m* is undefined, the line is vertical.

d. (-1,2), (1,0)

m = $\frac{-2}{2} = -1$

Because *m* < 0, the line falls.

Practice Exercises for Example 2

Try These:

Without graphing, tell whether the line through the given points rises, falls, is horizontal, or is vertical.

3. (–4, 2), (1,0) **Solution:** m = $\frac{2}{-5}$ falls

4. (1,6), (1,0) **Solution:** m = undefined vertical

5. (0,–3), (4,–3) **Solution:** m = 0 horizontal

6. (–1,–1), (2,0) **Solution:** m = $\frac{1}{3}$ rise

EXAMPLE 3

Classify parallel and perpendicular lines

Tell whether the lines are parallel, perpendicular, or neither.

Line 1: through (0, 2) and (1, 3)
Line 2: through (1, 0) and (2, –1)

The slope of Line 1 is 1. The slope of Line 2 is –1. Because $m_1 \cdot m_2$ is –1, m_1 and m_2 are negative reciprocals of each other so the lines are perpendicular.

Practice Exercises for Example 3

Try These:

Tell whether the lines are *parallel, perpendicular, or neither.*

	Solution	
7. Line 1: through (–1, –1) and (1, 3)	m = 2	
Line 2: through (–2, –2) and (1, 4)	m = 2	lines are parallel
8. Line 1: through (1, 5) and (0, 3)	m = 2	
Line 2: through (2, –3) and (0, 1)	m = -2	lines are neither parallel or perpendicular

EXAMPLE 4

Solve a multi-step problem

Use the table, which shows the growth of human hair over 4 months, to find the average rate of change in the length of human hair over time. Then predict the length of human hair at 9 months.

Month	1	2	3	4
Length of hair (in inches)	6	6.4	7	7.5

STEP 1

Find the average rate of change.

Average rate of change = $\frac{\text{change in length}}{\text{change in time}} = \frac{7.5 - 6}{4 - 1} = \frac{1.5}{3}$ = 0.5 inches per month

STEP 2

Predict the length of human hair at 9 months.

First, find the increase in the length of hair from 4 months to 9 months.

Increase in length = (5 months)(0.5 inch per month) = 2.5 inches

Now add this amount to the length of the hair at 4 months.

7.5 + 2.5 = 10

At 9 months, the length of human hair will be about 10 inches.

Practice Exercise for Example 4

Try This:

9. Use the average rate of change from Example 4 to predict the length of human hair at 11 months.

Solution: (7 months) (.5) = 3.5 inches

7.5 + 3.5 = 11 inches

At 11 months, the length of human hair will be about 11 inches.

Study Guide

Objective 4: <u>Graph linear equations in slope-intercept or standard form.</u>

Vocabulary

The **parent function** is the most basic function in a family.

A ***y*-intercept** of a graph is the point where the graph intersects the *y*-axis (when x = 0).

The equation $y = mx + b$ is said to be in **slope-intercept form**.

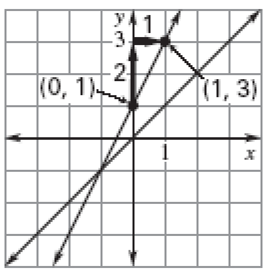
The **standard form of a linear equation** is $Ax + By = C$ where A and B are not both zero.

An ***x*-intercept** of a graph is the point where a graph intersects the x-axis (when y = 0).

EXAMPLE 1

Graph an equation in slope-intercept form

Graph the equation $y = 2x + 1$. Compare the graph with the graph of $y = x$.

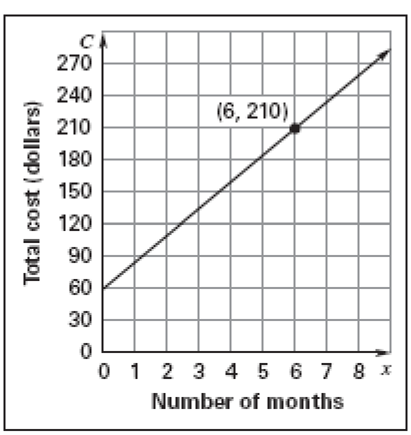


STEP 1 The equation is already in slope-intercept form.
STEP 2 Identify the *y*-intercept. Plot the point (0, 1).
STEP 3 Identify the slope. The slope is 2. Plot a second point by starting at (0, 1) and then moving up 2 units and right 1 unit. The second point is (1, 3).
STEP 4 Draw a line through the two points.
Comparison: The graph of $y = 2x + 1$ has a *y*-intercept of (0, 1), but the graph of $y = x$ has a *y*-intercept of (0, 0). And, the graph of $y = 2x + 1$ has a slope of 2, but the graph of $y = x$ has a slope of 1.

EXAMPLE 2

Solve a multi-step problem

A tennis club charges an initiation fee of $60 and a monthly fee of $25. Graph the equation $C = 25x + 60$ that represents the cost C of membership. Describe what the slope and *y*-intercept represent. Estimate the total cost after 6 months.



The slope represents the monthly fee and the *y*-intercept represents the initiation fee.

From the graph you can see that the total cost after 6 months is $210.

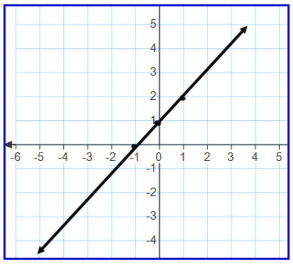
Practice Exercises for Examples 1 and 2

Try These:

Graph the equation. Compare the graph with the graph of *y* = *x*.

1. $y = x + 1$

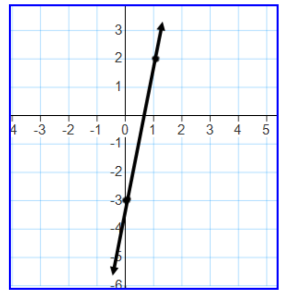
Solution:



The graph of $y = x + 1$ is translated up one.

2. $y = 5x - 3$

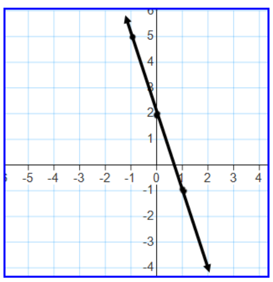
Solution:



The graph of y = 5x – 3 has a vertical stretch by a factor of 5 and is translated down 3.

3. $y = -3x + 2$

Solution:



The graph of $y = -3x + 2$ has a vertical stretch by a factor of 3, is reflected over the x-axis, and is translated up 2 units.

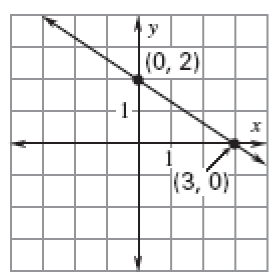
4. Rework Example 2 to find the total cost of your membership after 9 months if the initiation fee is \$50 and the monthly fee is \$20.

Solution: y = 20 x + 50
y = 20 (9) + 50
y = \$230

EXAMPLE 3

Graph an equation in standard form Ax + By = C

Graph 2*x* + 3y= 6.



STEP 1 The equation is already in standard form.

STEP 2 Identify the *x*-intercept.

$2x + 3(0) = 6$ Let $y = 0$.
$x = 3$ Solve for x.
The *x*-intercept is (3, 0). Plot (3, 0).

STEP 3 Identify the y-intercept.

$2(0) + 3.y = 6$ Let $x = 0$.
$y = 2$ Solve for y.
The *y*-intercept is (0, 2). Plot (0, 2).

STEP 4 Draw a line through the two points.

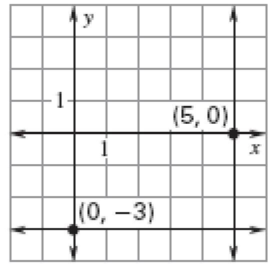
EXAMPLE 4

Graph horizontal and vertical lines

Graph the equation in a coordinate plane.

a. $y = -3$

b. $x = 5$



Solution

a. The graph of $y = -3$ is the horizontal line that passes through the point (0, –3).

b. The graph of $x = 5$ is the vertical line that passes through the point (5, 0)

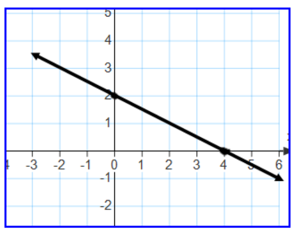
Practice Exercises for Examples 3 and 4

Try These:

Graph the equation.

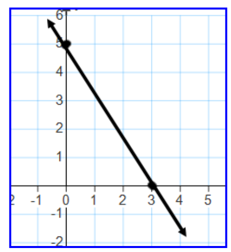
5. $x + 2y = 4$

Solution: intercepts (4, 0) (0, 2)



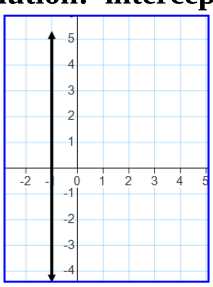
6. $5x + 3y = 15$

Solution: intercepts (3, 0) (0, 5)



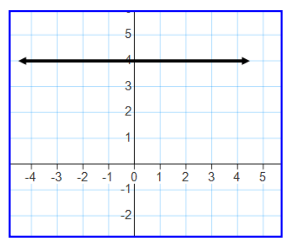
7. $x = -1$

Solution: intercept (0, -1)



8. $y = 4$

Solution: intercept (0, 4)



9. Find the x- and y-intercepts for the equation 3x - 2y = 8.

Solution: $(\frac{8}{3}, 0)$ and (0, -4)

Study Guide

Objective 5: Write linear equations.

Vocabulary

The **point-slope form** of the equation of a line is given by $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

EXAMPLE 1

Write the equation of a line given the slope and a point

a. Write the equation of the line with a slope of –2 and y-intercept of (0, 3).

Solution

a. $y = mx + b$	Use slope-intercept form.
$y = -2x + 3$	Substitute –2 for m and 3 for b.

b. Write an equation with a slope of 4 that passes through the point (– 1, 7).

Solution

b. $y - y_1 = m(x - x_1)$	Use point-slope form.	**OR**	Use slope-intercept form.	y = mx + b
$y - 7 = 4(x + 1)$	Substitute for m, x_1 and y_1		Substitute for y, m, and x	7 = (4)(-1) + b
$y - 7 = 4x + 4$	Distributive property		Solve for b.	b = 11
$y = 4x + 11$	Write in slope-intercept form.		Write in slope-intercept form.	So y = 4x + 11

EXAMPLE 2

Write equations of parallel or perpendicular lines

Write an equation of the line that passes through (0, 1) and is (a) parallel to, and (b) perpendicular to, the line y= 2x+ 7.

Solution

a. The given line has a slope of $m_1 = 2$. A line parallel to the given line has a slope of $m_2 = 2$. Use the point (0, 1) and $m_2 = 2$ to write the equation of the line.

$y - y_1 = m(x - x_1)$	Use point-slope form.	**OR**	Use slope-intercept form.	y = mx + b
$y - 1 = 2(x + 0)$	Substitute for m, x_1 and y_1		Substitute for y, m, and x	1 = (2)(0) + b
$y - 1 = 2x$	Distributive property		Solve for b.	b = 1
$y = 2x + 1$	Write in slope-intercept form.		Write in slope-intercept form.	So y = 2x + 1

b. A line perpendicular to a line with slope $m_1 = 2$ must have a slope of $m_2 = -\frac{1}{m_1} = -\frac{1}{2}$ Use the point (0, 1) and m_2 to write the equation of the line.

$y - y_1 = m(x - x_1)$	Use point-slope form.	**OR**	Use slope-intercept form.	y = mx + b
$y - 1 = -\frac{1}{2}(x - 0)$	Substitute for x_1 and y_1		Substitute for y, m, and x	1 = (-1/2)(0) + b
$y - 1 = -\frac{1}{2}x - \left(-\frac{1}{2}\right)(0)$	Distributive property		Solve for b.	b = 1
$y = -\frac{1}{2}x + 1$	Write in slope-intercept form.		Write in slope-intercept form.	$y = -\frac{1}{2}x + 1$

Practice Exercises for Examples 1 and 2

Try These:

Write an equation of the line with the given conditions. **Solution:**

1. With a slope of 4 and a y-intercept of –1 — $y = 4x - 1$
2. With a slope of –5 that passes through the point (3, –2) — $y = -5x + 13$
3. That passes through (2, 3) and parallel to $y = -x + 3$ — $y = -x + 5$
4. That passes through (0, 5) and perpendicular to $y = 4x + 1$ — $y = -\frac{1}{4}x + 5$

EXAMPLE 3

Write the equation of a line given two points

Write the equation of the line that passes through (2, 8) and (4, 14).

Find the slope of the line through $(x_1, y_1) = (2, 8)$ and $(x_2, y_2) = (4, 14)$.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14-8}{4-2} = \frac{6}{2} = 3$$

Use the point (2, 8) and $m = 3$ to write the equation.

$y - y_1 = m(x - x_1)$	Use point-slope form.	**OR**	Use slope-intercept form.	y = mx + b
$y - 8 = 3(x - 2)$	Substitute for m, x_1 and y_1		Substitute for y, m, and x	8 = (2)(3) + b
$y - 8 = 3x - 6$	Distributive property		Solve for b.	b = 2
$y = 3x + 2$	Write in slope-intercept form.		Write in slope-intercept form.	So y = 3x + 2

Practice Exercises for Example 3

Try These: Write the equation of the line that passes through the given points.

Solutions:

5. (1,–1), (4, 2) — $y = x - 2$
6. (–2, 4), (3,–1) — $y = -x + 2$
7. (–3,–1), (0, 1) — $y = \frac{2}{3}x + 1$

9. Find the slope of the line passing through the points (6,4) and (-1,2). — $m = \frac{2}{7}$

10. Write the equation of the line passing through the point (-1,3) with a slope of $-\frac{2}{5}$. — $y = -\frac{2}{5}x + \frac{13}{5}$

11. Write the equation of the line that passes through the point (0,4) and is perpendicular to the line $y = -\frac{5}{4}x$. — $y = \frac{4}{5}x + 4$

**Now complete Assignment 2.

Study Guide

Objective 6: Solve linear inequalities in two variables.

The solutions to a linear inequality in two variables can be graphed on an x-y coordinate plane.

When graphing linear inequalities in two variables you will want to:

1. Get the line in slope intercept form.
2. Graph the y- intercept and use the slope to get one or more points.
3. Connect the points making a solid or dashed line.
4. Shade above or below the line.

	line		shading	
	Solid	dashed	above	below
≤	x			x
≥	x		x	
<		x		x
>		x	x	

Note: horizontal lines and vertical lines are special cases where you will shade above or to the right if you symbol is ≥ or >, and below or to the left if you symbol is ≤ or <.

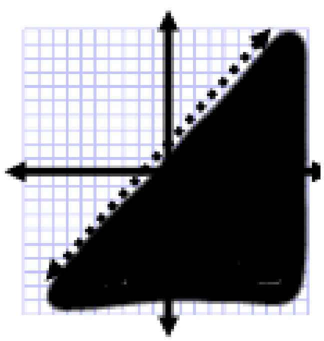
EXAMPLE 1

Graph the linear inequality

Graph $y < x + 2$

Solution

Since the line is in slope-intercept form you can graph the y-intercept (0, 2) and use the slope of 1 to get more points on the line. Since the symbol is < you will use a dashed line and shade below the line.



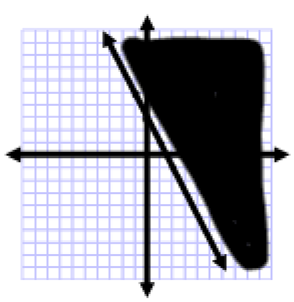
EXAMPLE 2

Graph the linear inequality

Graph $y \geq -2x + 3$

Solution

Since the line is in slope-intercept form you can graph the y-intercept (0, 3) and use the slope of -2 to get more points. Since the symbol is ≥ you will use a solid line and shade above the line.



EXAMPLE 3

Graph the linear inequality

Graph $-2x - 3y \leq 3$

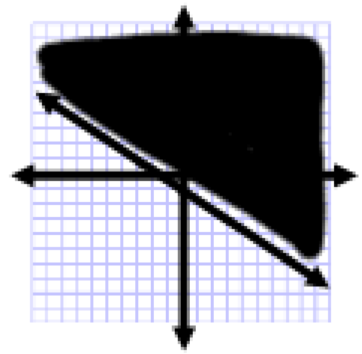
Solution

Since the line is not in slope-intercept form, solve for y in terms of x first.

*REMEMBER if you multiply or divide by a negative number while solving for y reverse your inequality symbol.

$y \geq -\frac{2}{3}x - 1$

Now graph using the y-intercept (0, -1) and use the slope $-\frac{2}{3}$ to get more points. Since the symbol is ≥ you will use a solid line and shade above the line.



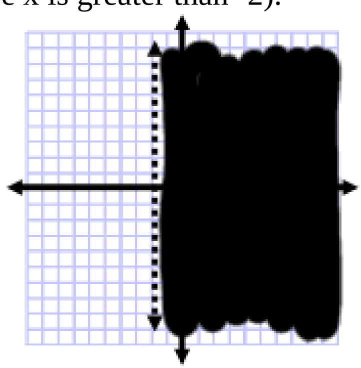
EXAMPLE 4

Graph the linear inequality

Graph $x > -2$

Solution

The graph of $x > -2$ is a vertical line. Since the symbol is > you will use a dashed line and shade to the right of the line (all the points where x is greater than -2).



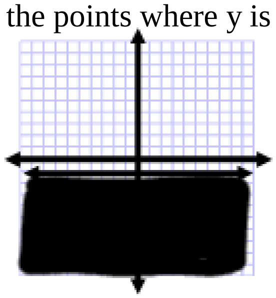
EXAMPLE 5

Graph the linear inequality

Graph $y \leq -1$

Solution

The graph of $y \leq -1$ is a horizontal line. Since the symbol is ≤ you will use a solid line and shade below the line (all the points where y is less than -1).



**Now complete Assignment 3.

Study Guide

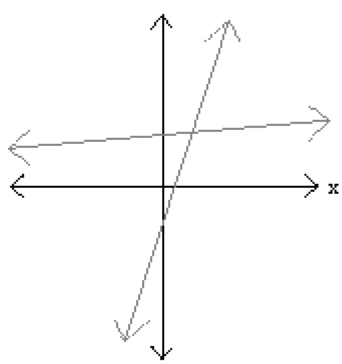
Objective 7: <u>Solving Systems of Equations</u>

Vocabulary

The **solution to a system** of equations or is the point(s) that the two equations have in common (their intersection).

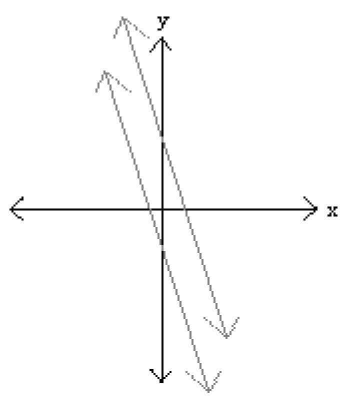
There are three possibilities for the way two linear equations could meet. They could intersect once, none (they lines are parallel), or at an infinite number of points (the lines are the same – they coincide – so they share every point on the line).

A system is said to be **consistent** if there exists at least one solution – the lines intersect once or are the same line.
A system is said to be **inconsistent** if there are no solutions – the lines are parallel.
A system is said to be **independent** if the two lines intersect; therefore have one solution.
A system is said to be **dependent** if the two lines are the same – they coincide and have infinitely many solutions.



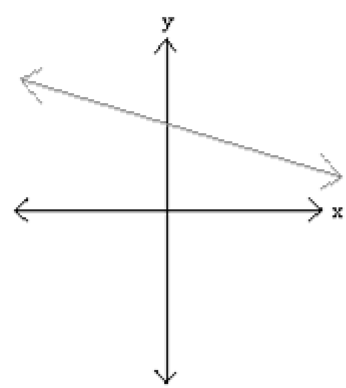
Consistent
Independent

the lines intersect
one solution
different y-int, different slope



Inconsistent

the lines are parallel
no solution
different y-int, same slope



Consistent
Dependent

the lines coincide
infinite number of solutions
same y-int, same slope

Method 1: Solving by Graphing
Step 1: put both equations in slope-intercept form
Step 2: graph both lines
Step 3: identify the point or points that the graphs have in common (their intersection point)

EXAMPLE 1

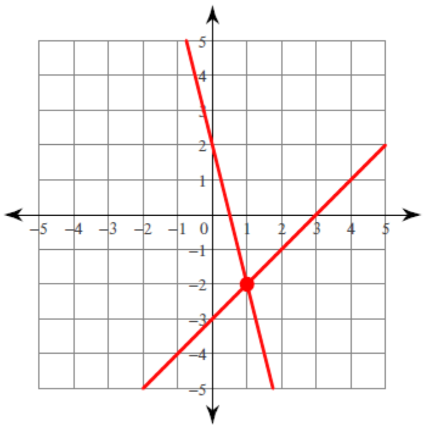
Solve a system of linear equations by graphing

Solve

$4x + y = 2$
$x – y = 3$

Solution

$y = -4x + 2$	Put both equations in slope-intercept form.
$y = x – 3$	Graph the equations.



The solution to the system is the point of intersection. (1, -2)

Method 2: Solving by Substitution

Step 1: isolate one of the variables in one of the equations
Step 2: substitute into the **other** equation
Step 3: solve
Step 4: plug the value back in to find the value of the other variable.
Step 5: write your solution as a point.

EXAMPLE 2

Solve a system of linear equations by substitution

Solve

$$\begin{cases} 5x + 2y = -9 \\ y - 3x = 12 \end{cases}$$

Solution

$y = 3x + 12$	The easiest variable to isolate is the y in the second equation.
$5x + 2(3x +12) = -9$	Substitute into the other equation.
$5x + 6x + 24 = -9$	Simplify and solve.
$11x = -33$	
$x = -3$	This is the x-value of the point of intersection
$y = 3(-3) + 12$	Plug back in to find the y-value.
$y = 3$	This is the y-value of the point of intersection
(-3, 3)	This is the solution to the system.

EXAMPLE 3

Solve a system of linear equations by substitution

Solve

$$\begin{cases} 6x - y = 26 \\ 3x - \frac{1}{2}y = 13 \end{cases}$$

Solution

y = 6x – 26	The easiest variable to isolate is the y in the first equation.
3x – ½(6x – 26) = 13	Substitute into the other equation.
3x – 3x + 13 = 13	Simplify and solve.
13 = 13	The variables cancel and result in a true statement.
	There are infinite solutions.
	Conclude that the lines coincide.
$\{(x, y) \mid 6x - y = 26\}$	The solutions to the system are all the points on the line 6x – y = 26.

Method 3: Solving by Elimination

Step 1: arrange the equations so that the x and y terms are lined up on the same side in both equations.
Step 2: look for a pair of opposites. If there aren't any opposites, multiply one or both equations to create a pair of opposites. It doesn't matter which of the variables is the opposite pair.
Step 3: add the equations together – the opposite pair should eliminate.
Step 4: solve.
Step 5: plug the value back in to find the value of the other variable.
Step 6: write your solution as a point.

EXAMPLE 4

Solve a system of linear equations by elimination

Solve

$$\begin{cases} x - 9y = -13 \\ 2x + y = -7 \end{cases}$$

Solution

x – 9y = -13 → x – 9y = -13	There are no opposite pairs. The easiest way to make
(2x + y = -7)(9) → 18x + 9y = -63	opposites would be to multiply the second equation by 9 to
	make the opposite pair -9y, 9y

x – 9y = -13	
<u>18x + 9y = -63</u>	Add down.
19x = -76	Solve.
x = - 4	This is the x-value of the point of intersection
-4 – 9y = -13	Plug back in to find the y-value.
-9y = -9	Solve.
y = 1	This is the y-value of the point of intersection
(-4, 1)	This is the solution to the system.

EXAMPLE 5

Solve a system of linear equations by elimination

Solve

$$\begin{cases} 2x - 3y = 7 \\ \frac{2}{3}x - y = 9 \end{cases}$$

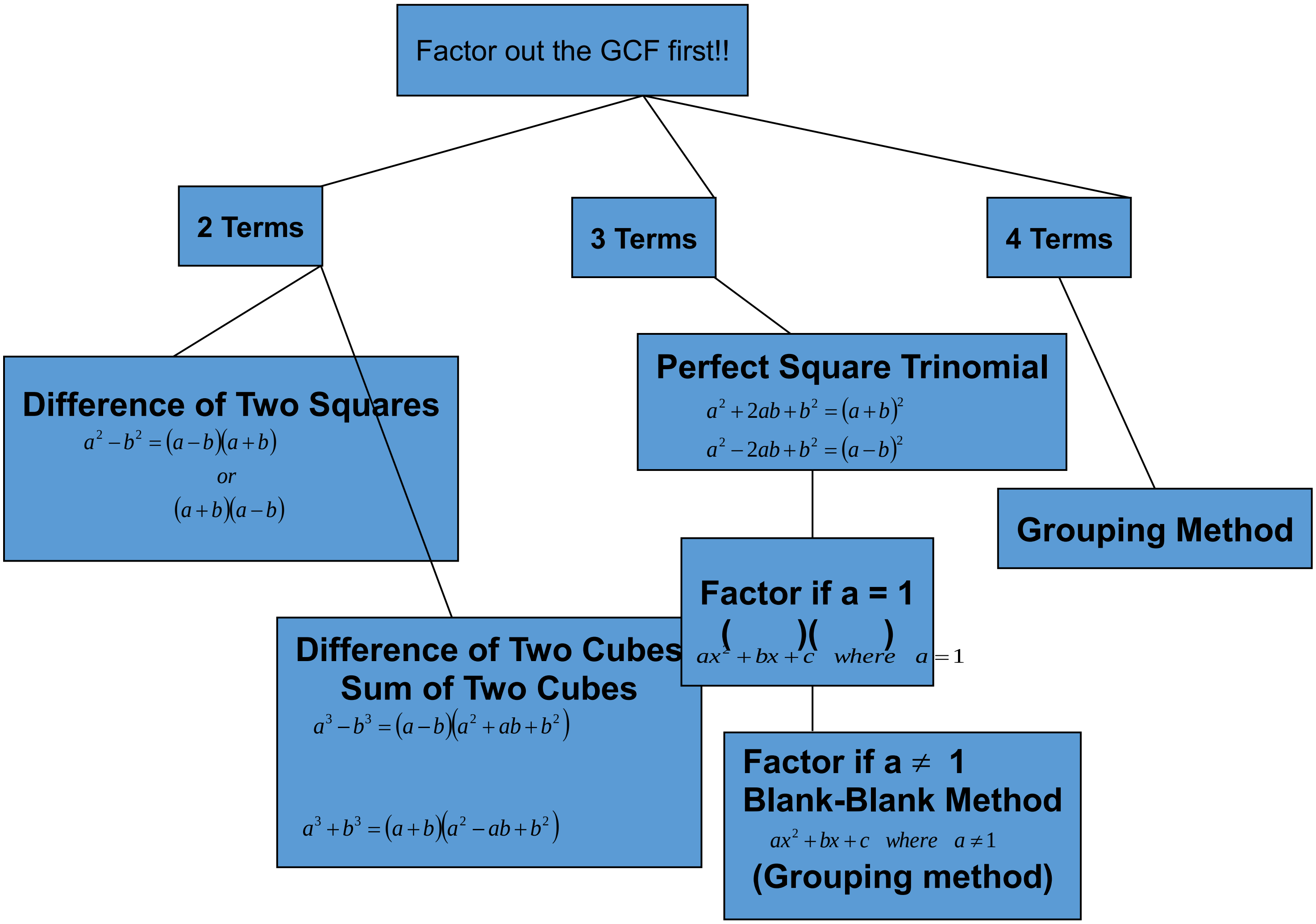
Solution

$2x - 3y = 7$	$\rightarrow$ $2x - 3y = 7$	There are no opposite pairs. The easiest way to make opposites would be to multiply the second equation by -3 to make the opposite pair -3y, 3y
$(\frac{2}{3}x - y = 9)(-3)$	$\rightarrow$ $-6x + 3y = -27$	

$2x - 3y = 7$	
$\underline{-6x + 3y = -27}$	Add down.
$0 = -20$	Solve.
$0 \neq -20$	The variables cancel and result in a false statement.
Ø	There are no solutions.
	Conclude that the lines are parallel, so there is no solution.

**Now complete Assignment 4.

ALGEBRA
REVIEW OF FACTORING



Study Guide

Objective 8: <u>Factoring Expressions</u>

Factoring is the reverse of multiplying.

Multiplying →	Multiplying →
$2x(x+3) = 2x^2 + 6x$	$(x + 5)(x + 2) = x^2 + 7x + 10$
← Factoring	← Factoring

***GCF**

To factor an expression containing two or more terms, factor out the GCF.

1.) Find GCF
2.) Express polynomial as product of the GCF and its remaining factors

EXAMPLE 1

Factor each expression

1. $20x^4 + 36x^2$

Solution:

$4x^2(5x^2 + 9)$ — $4x^2$ is the GCF for both terms

2. $3x(4x + 5) – 5(4x + 5)$

Solution:

$(4x + 5)(3x – 5)$ — $(4x + 5)$ is the GCF for both terms

3. $5x^3 - 10x^2 + 25x + 50$

Solution:

$5(x^3 - 2x^2 + 5x + 10)$ — 5 is the GCF for all 4 terms

***Grouping Method**

Use the grouping method to factor polynomials with four terms

1.) Group terms that have common factors
(do not group 1st term w/last term)
2.) Factor the GCF from each binomial
3.) Binomial factors must match
4.) Express answer as a product of the two binomials

EXAMPLE 2

Factor each Quadratic Expression

4. $15x^2 - 3x + 10x - 2$

Solution

$(15x^2 - 3x)(+10x - 2)$	Group the first two and the last two terms
3x(5x – 1) + 2(5x – 1)	Factor out the GCF of each pair
(5x – 1)(3x + 2)	Factor out the GCF one last time (matching parenthesis)

5. $8x^2 - 5x - 24x + 15$

Solution

$(8x^2 - 5x)(-24x + 15)$	Group
x(8x – 5) – 3(8x – 5)	Factor out the GCF of each pair two times (careful with signs)
(8x – 5)(x – 3)	Factor out the GCF one last time (matching parenthesis)

***Difference of Two Squares**

$$a^2 - b^2 = (a+b)(a-b) = (a-b)(a+b)$$

EXAMPLE 3

Factor each Quadratic Expression

6. $4x^2 - 25$ (Both terms in the binomial are perfect squares and they are being subtracted)

Solution

(2x + 5)(2x – 5)

7. $36x^2 - 81$ (Both terms in the binomial are perfect squares and they are being subtracted)

Solution

(6x + 9)(6x – 9)

***Factoring Perfect-Square Trinomials**

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^2 - 2ab + b^2 = (a-b)^2$$

1.) Is the first term a perfect square?
2.) Is the last term a perfect square?
3.) Is the middle term equal to twice (positive or negative) the product of the square root of the first term and the square root of the last term?

EXAMPLE 4

Factor each Quadratic Expression

8. $9x^2 - 42x + 49$

Solution

$-2(3x)(7) = -42x$ First, check that you have a perfect square trinomial

$(3x-7)^2$ Factor (sign from the middle)

9. $16x^2 + 72x + 81$

Solution

$2(4x)(9) = 72x$

$(4x+9)^2$

***Factoring $ax^2 + bx + c$ where a = 1 using integers**.

$$x^2 + bx + c = (x + r)(x + s)$$

Quadratic trinomials of the form $ax^2 + bx + c$ can be factored using integers if and only if there are two integers whose product is *ac* and whose sum is *b*.

Find two integers *r* and *s* whose product equals the last term *(c)* and whose sum equals the coefficient of the middle term *(b).*

EXAMPLE 5

Factor each Quadratic Expression (leading coefficient of 1)

	Solution	
10. $x^2 + 5x + 6$	$(x + 2)(x + 3)$	look for numbers whose product is 6 and sum is 5
11. $x^2 - 7x + 10$	$(x - 5)(x - 2)$	look for numbers whose product is 10 and sum is -7
12. $x^2 + 3x - 10$	$(x + 5)(x - 2)$	look for numbers whose product is -10 and sum is 3
13. $x^2 - 7x - 30$	$(x - 10)(x + 3)$	look for numbers whose product is -30 and sum is -7

***Factoring $ax^2 + bx + c$ where $a \neq 1$ using integers**.

1.) Find two integers whose product is equal to the product of the first and last terms *(ac),* and whose sum equals the coefficient of the middle term *(b).*
2.) Rewrite the middle term using the two integers.
3.) Use the grouping method and express your answer as a product of two binomials.

EXAMPLE 6

Factor each Quadratic Expression (leading coefficient ≠ 1)

14. $8x^2 - 14x + 3$

Solution

$8x^2 - 12x - 2x + 3$ Rewrite the middle term with numbers that multiply to 24 and add to -14

$(8x^2 - 12x)(-2x + 3)$ Treat as a grouping problem

4x(2x -3) -1(2x – 3) GCF for both binomials

(2x – 3)(4x – 1) GCF one last time - matching factors of (2x – 3)

15. $3x^2 + 11x - 20$

Solution

$3x^2 + 15x - 4x - 20$ Rewrite the middle term with numbers that multiply to -60 and add to 11

$(3x^2 + 15x)(-4x - 20)$ Treat as a grouping problem

3x(x + 5) - 4(x + 5) GCF for both binomials

(x + 5)(3x – 4) GCF one last time - matching factors of (x + 5)

**Now complete Assignment 5.

Study Guide

Objective 9: <u>Solving Quadratic Equations</u>

Methods for Solving Quadratic Equations

Quadratics equations are of the form $ax^2 + bx + c = 0, \ where \ a \neq 0$

Quadratics may have two, one, or zero real solutions.

1. FACTORING

Set the equation equal to zero. If the quadratic side is factorable, factor, then set each factor equal to zero.

<u>Example:</u> $x^2 = -5x - 6$

Move all terms to one side $x^2 + 5x + 6 = 0$

Factor $(x+3)(x+2) = 0$

Set each factor to zero and solve $x + 3 = 0 \qquad x + 2 = 0$

$x = -3 \qquad x = -2$

2. PRINCIPLE OF SQUARE ROOTS

If the quadratic equation involves a **<u>SQUARE</u>** and a **<u>CONSTANT</u>** (no first degree term), position the square on one side and the constant on the other side. Then take the square root of both sides. (Remember, you cannot take the square root of a negative number, so if this process leads to taking the square root of a negative number, there are no real solutions.)

<u>Example 1:</u> $x^2 - 16 = 0$

Move the constant to the right side $x^2 = 16$

Take the square root of both sides $\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4, \ which \ means \ x = 4 \ and \ x = -4$

<u>Example 2:</u> $2(x+3)^2 - 14 = 0$

Move the constant to the other side $2(x+3)^2 = 14$

Isolate the square $(x+3)^2 = 7$ (divide both sides by 2)

Take the square root of both sides $\sqrt{(x+3)^2} = \pm\sqrt{7}$

$x + 3 = \pm\sqrt{7}$

Solve for x $x = -3 \pm \sqrt{7}$

This represents the exact answer.

Decimal approximations can be found using a calculator.

3. COMPLETING THE SQUARE

If the quadratic equation is of the form $ax^2 + bx + c = 0,\ where\ a \neq 0$ and the quadratic expression is not factorable, try completing the square.

Example: $x^2 + 6x - 11 = 0$

****Important: If $a \neq 1$, divide all terms by "a" before proceeding to the next steps.**

Move the constant to the right side and supply a blank on each side: $x^2 + 6x + \_\_\_\_\_ = 11 + \_\_\_\_\_$

Find half of b, which means $\frac{b}{2}$: $\frac{6}{2} = 3$

Square half of b: $\left(\frac{b}{2}\right)^2$: $3^2 = 9$

Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation: $x^2 + 6x + 9 = 11 + 9$

Factor the quadratic side (which is a perfect square because you just made it that way!): $(x+3)(x+3) = 20$

Then write in perfect square form: $(x+3)^2 = 20$

Take the square root of both sides: $\sqrt{(x+3)^2} = \pm\sqrt{20}$

$x + 3 = \pm\sqrt{20}$

Solve for x

$x = -3 \pm \sqrt{20}$ *Simplify the radical*

$x = -3 \pm 2\sqrt{5}$

This represents the exact answer.
Decimal approximations can be found using a calculator: 1.472 and -7.472

4. QUADRATIC FORMULA

Any quadratic equation of the form $ax^2 + bx + c = 0,\ where\ a \neq 0$ can be solved for both real and imaginary solutions using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $x^2 + 6x - 11 = 0 \quad (a = 1,\quad b = 6,\quad c = -11)$

Substitute values into the quadratic formula:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-11)}}{2(1)} \rightarrow x = \frac{-6 \pm \sqrt{36 + 44}}{2} \rightarrow x = \frac{-6 \pm \sqrt{80}}{2} \quad \textit{simplify the radical}$$

$$x = \frac{-6 \pm 4\sqrt{5}}{2} \rightarrow \frac{-6}{2} \frac{\pm 4\sqrt{5}}{2} \rightarrow x = -3 \pm 2\sqrt{5} \quad \textit{This is the final simplified EXACT answer}$$

**Now complete Assignment 6.

Assignment 1: Objectives 1 and 2 Solving Equations and Inequalities

Solve each equation.

1. $4x - 7 = 4(x - 2) - 2$

2. $0.05(x - 5) - 0.08x = 1.01$

3. $-4(2x - 6) = 5x + 24 - 7x$

4. $4(x + 3) - 4 = 8(\frac{1}{2}x + 1)$

5. $7(2x - 3) - 4(3x - 5) = 4 + 6x - 13$

6. $\frac{y+10}{6} + \frac{y-6}{9} = \frac{y+2}{2}$

Solve and graph.

7. $2(1-x)+5 \leq 3(2x-1)$

8. $-14 < -7(3x+2)$

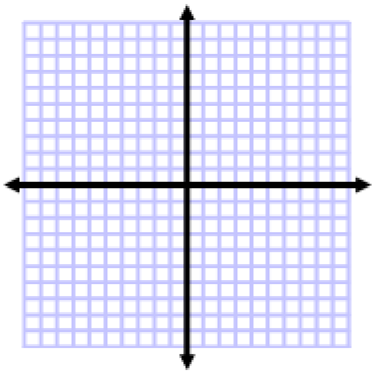
9. $\frac{2x}{3}+12 < \frac{x}{6}+18.$

10. $\frac{2}{3} \geq \frac{2x-3}{12}$

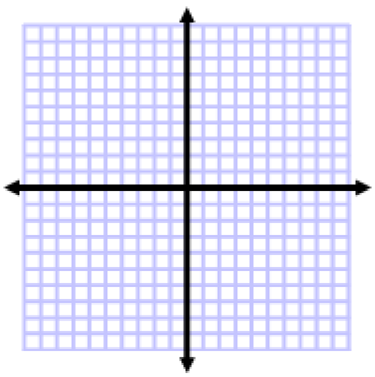
Assignment 2: Objectives 3, 4, and 5 Finding slope, graphing, and writing linear equations

Part I – Graph each of the following carefully.

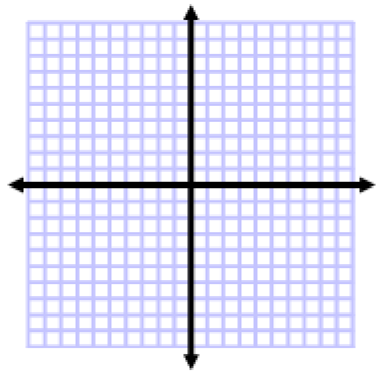
1. 3x + 2y = 12



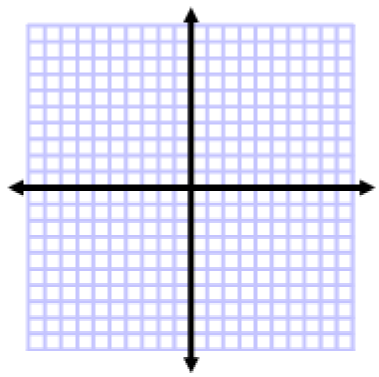
2. x = – 3



3. y = 5



4. 2x – y = 5



Part II – Find the x and y intercepts for the following linear graphs.

5. 2x + 5y = 10

6. 3x = y

Part III - Find the slope of the line passing through the following points:

7. (2, 8) and (–3, 8)

8. (5, –4) and (–2, 7)

Part IV - Write the equation of the line that meets the following criteria:

9. Passing through (5, –2) with a slope of $\frac{2}{5}$.

10. Perpendicular to the graph $2x - 5y = 3$ and passing through (–2, 7).

11. Find the equation of the line through (6,1) and (-2,5).

12. Write the equation of the line passing through the point (8,4), parallel to the line $y = \frac{-5}{4}x$.

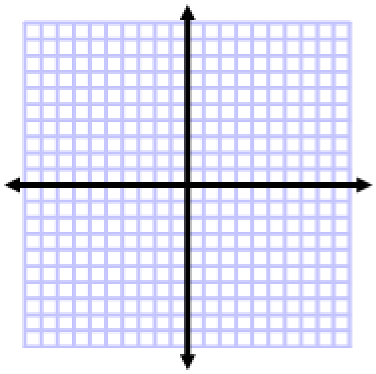
13. Write the equation of the line where $m = 0$ and the y-intercept is (0,4).

14. Write the equation of the line that pass through (12,-3) and (12,6).

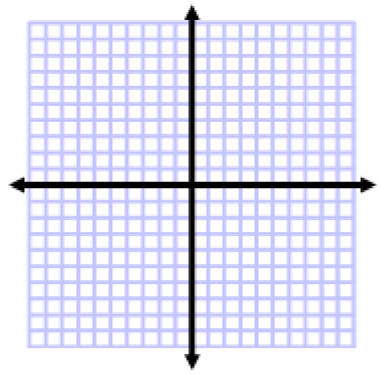
15. Determine the slope of the line $5x + 2y = 20$. Describe how the line will be positioned (rise, fall, horizontal, or vertical).

Part V - Graph each equation by finding the x- and y-intercepts.

16. $3x + 2y = 12$



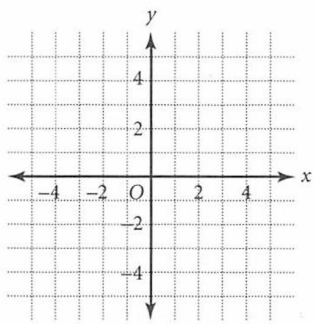
17. $x - 3y = -6$



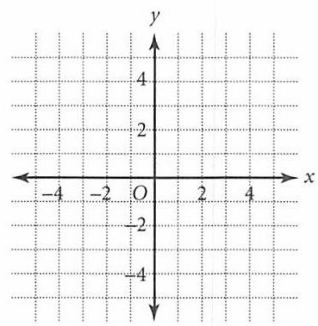
Assignment 3: Objective 6 Solving linear inequalities in two variables

Graph each linear inequality.

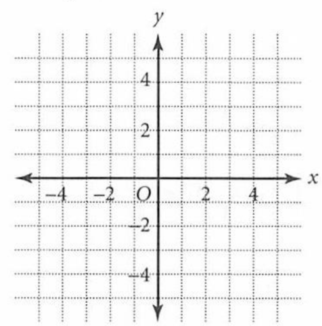
1. $y < -x$



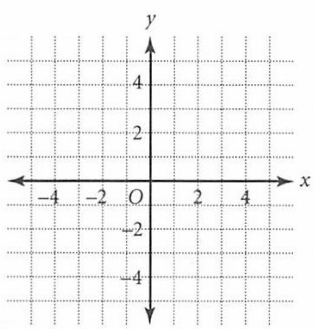
2. $y \geq -3x - 2$



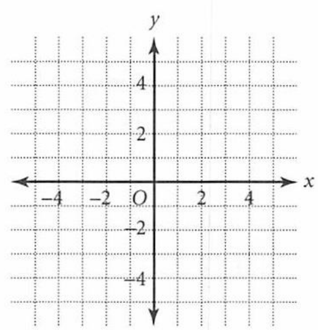
3. $y > \frac{1}{4}x + 1$



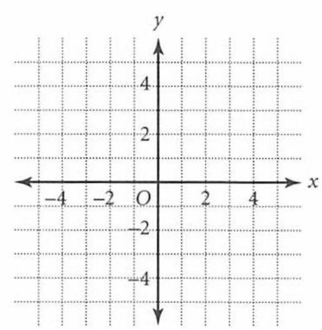
4. $2x + y \leq -3$



5. $x - y > 4$



6. $x \geq -1.5$



7. Sheila earns a basic wage of $8 per hour. Under certain conditions, she is paid $12 per hour. The most that she can earn in one week is $400.

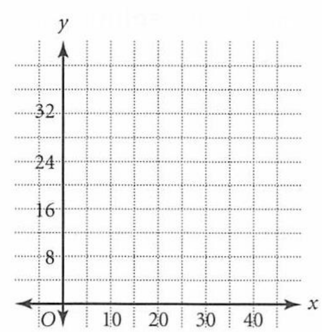
a. Write an inequality that describes her total weekly wages for *x* hours at $8 per hour and for *y* hours at $12 per hour.

____

b. Graph the inequality on the grid at right.

c. What is the maximum number of hours that Sheila can work for $8 per hour? for $12 per hour?

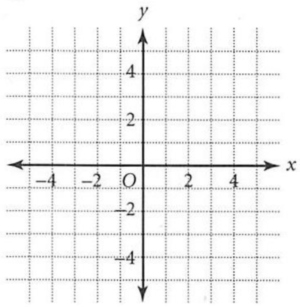
____



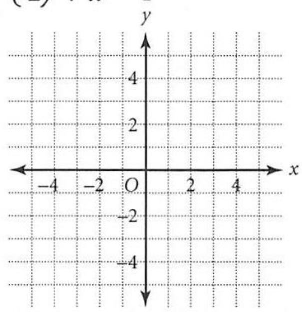
Assignment 4: Objective 7 Solving Systems

Graph and classify each system. Then find the solution from the graph.

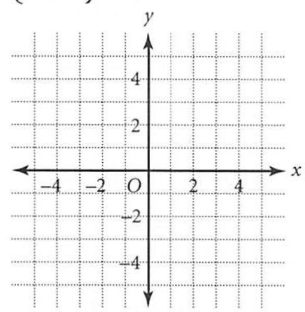
1. $\begin{cases} y = x + 4 \\ y + x = 4 \end{cases}$



2. $\begin{cases} \frac{1}{2}x + y = 2 \\ 2y + x = 4 \end{cases}$



3. $\begin{cases} 3x + 4y = -7 \\ 2x + y = -3 \end{cases}$



Use substitution to solve each system of equations. Check your solution.

4. $\begin{cases} y + 2x = 11 \\ x + y = 5 \end{cases}$

5. $\begin{cases} x - 10y = 2 \\ x - 6y = 6 \end{cases}$

6. $\begin{cases} 8x = y \\ 2x + y = 5 \end{cases}$

Use elimination to solve each system of equations.
Check your solution.

7. $\begin{cases} 5x + 9y = -7 \\ 2x + 3y = -1 \end{cases}$

8. $\begin{cases} \frac{2}{3}x - 3y = \frac{1}{5} \\ 2x - 9y = 4 \end{cases}$

9. $\begin{cases} 7y - x = 8 \\ x - y = 4 \end{cases}$

Use any method to solve each system of linear equations. Check your solution.

10. $\begin{cases} 12x - y = 2 \\ 4x + 3y = 4 \end{cases}$ ____________

11. $\begin{cases} y = 6x \\ 2x + 5y = 16 \end{cases}$ ____________

12. $\begin{cases} 4x + y = 9 \\ 2y = -8x + 18 \end{cases}$ ____________

Assignment 5: Objective 8 Factoring Completely

Factor completely.

1. $x^2 + 12x + 36$

4. $4t^2 + 4t - 15$

2. $x^2 - 144$

5. $-9x^2 + 27x$

3. $b^2 - b + 2b - 2$

6. $9a^3b - 27ab$

7. $16x^2 - 81$

12. $a^3 + 9a^2 + 18a$

8. $9x^2 + 24x + 16$

13. $5x^2 - 20x + 10$

9. $6x^3 + 18x^2 - 5x - 15$

14. $2xy - x^2y + 6 - 3x$

10. $4x^2 + 25$

15. $3a^2 - 10a + 8$

11. $6x^2 - 7xy + 2y^2$

16. $x^2 + 2x - 63$

Assignment 6: Objective 9 Solving Quadratic Equations

Solve each equation by factoring.

1. $5x^2 + 25x = -30$

2. $x^2 = 3x$

3. $3x^2 - 7 = -20x$

4. $3x^2 = 1 + 2x$

Solve each equation by taking square roots.

5. $7x^2 - 1 = 671$

6. $7x^2 + 2 = 254$

7. $64x^2 + 1 = 17$

8. $8x^2 + 10 = 810$

Solve each equation by completing the square.

9. $8x^2 + 16x - 64 = 0$

10. $x^2 - 20x + 28 = 0$

11. $x^2 - 18x + 77 = 0$

12. $2x^2 + 8x - 24 = 0$

Solve each equation with the quadratic formula.

13. $3x^2 = 27$

14. $5x^2 - 6 = x$

15. $x^2 = 84 + 5x$

16. $4x^2 = 2 - 10x$