

# West Essex Regional School District

## AP Calculus AB Summer Assignment 2023-2024

Calculus AB covers the equivalent of a one semester college calculus course. Our focus will be on differential and integral calculus of a single variable and applications. You must take the exam to receive AP credit for this course.

### **A strong knowledge of the following material is required for this course:**

- Graphs of functions: linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions
- Language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on)
- Composite functions: definition, identity property, decomposing
- Properties of logarithms and exponents, using logarithms to solve exponential equations, using exponentiation to solve logarithmic equations, and using logarithms to expand equations.
- Factoring equations to solve for zeros.
- Simplifying rational functions.
- Using limits to describe asymptotes and end behavior of functions.
- Conditions for inverses, finding inverses of functions and properties of inverses.
- Unit circle: know the values of the trigonometric functions at the numbers  $0, \pi/6, \pi/4, \pi/3, \pi/2$ , and their multiples.
- Trigonometric ratios, inverse trigonometry, reciprocal ratios, and Pythagorean identity
- Defining limits and using limit notation.
- Determining limits from graphs, tables and algebraically.

### **Review**

- The following problems are indicative of the type that you will encounter this year. Use these problems as a study guide. You will be quizzed on these topics during the first week of school.
- If you don't remember how to do a problem, there are plenty of resources online, such as Khan Academy and Youtube (patrickjmt is great). It is your responsibility to be prepared for the first day of class.

### **Do NOT use a calculator.**

### **Supplies for Next Year**

The course and topics are cumulative so you are expected to retain and apply all knowledge acquired from the beginning of class. You need to stay organized with class materials that will include warm ups, check your understanding quizzes, guided notes, classwork and homework. You should have a 3-ring binder.

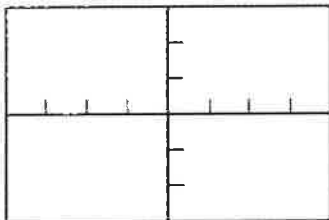
Join the summer Google Classroom to post questions and to check your answers: \_\_\_\_\_

# West Essex High School AP Calculus AB Summer 2023 Assignment

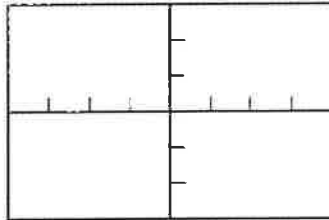
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1. Sketch each function. Note key characteristics (intercepts, asymptotes). Clearly indicate the units on the axes provided.

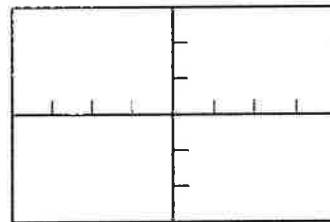
$$f(x) = x^2$$



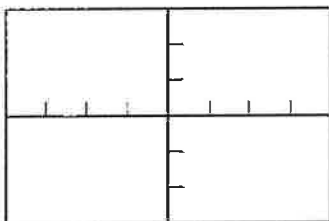
$$f(x) = x^3$$



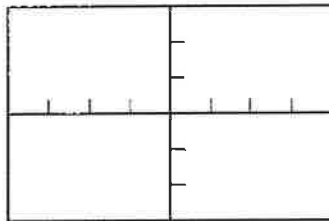
$$f(x) = |x|$$



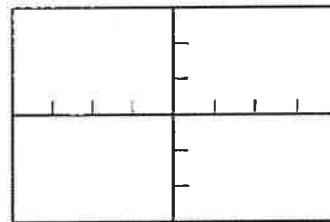
$$f(x) = \sin x$$



$$f(x) = \cos x$$



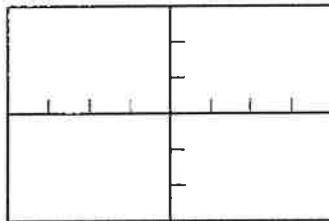
$$f(x) = \tan x$$



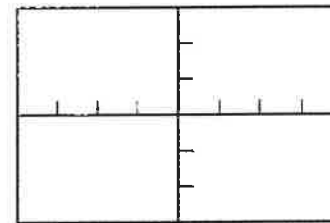
$$f(x) = \sec x$$



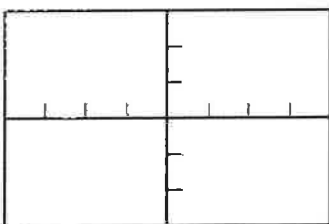
$$f(x) = 2^x$$



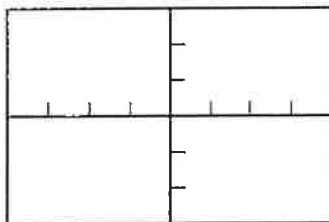
$$f(x) = e^x$$



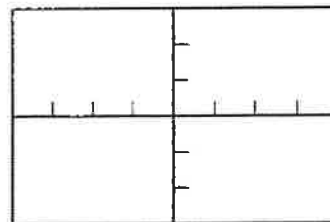
$$f(x) = \frac{1}{x}$$



$$f(x) = \sqrt{x}$$

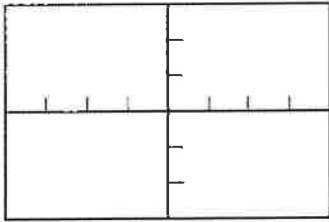


$$f(x) = \sqrt{4 - x^2}$$

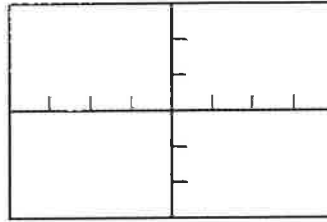


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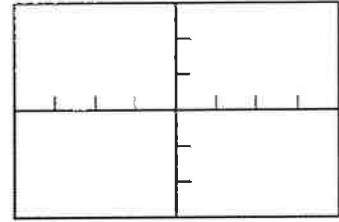
$$f(x) = \frac{|x|}{x}$$



$$f(x) = \frac{1}{x^2}$$



$$f(x) = \ln x$$



2. Find the domain for each function.

$$f(x) = \sqrt{4x - 3}$$

$$g(x) = \frac{x^2 - 2}{x - 3}$$

$$h(x) = \ln(x + 4)$$

3. For the function  $f(x) = x - 2$  and  $g(x) = \frac{x+5}{3}$ , find  $g(f(x))$
4. For the function  $f(x) = \cos x$  and  $g(x) = \ln x$ , find  $f(g(x))$
5. If  $k(x) = f(g(x))$ , for the function  $k(x) = e^{\sin x}$ , find  $f(x)$  and  $g(x)$
6. If  $k(x) = f(g(x))$ , for the function  $k(x) = (x + 10)^7$ , find  $f(x)$  and  $g(x)$
7. Solve for  $x$ .  

$$e^{\ln(2x+1)} = 5x$$
8. Solve for  $x$ .  

$$\frac{e^{2x+2}}{e^x} = 4$$
9. Solve for  $y = f(x)$ .  

$$\ln(y - 2) = x^2 + 6$$
10. Solve for  $x$ .  

$$e^{[3 \ln x - \ln(x^3 - 2x + 4)]} = 1$$
11. Find the  $x$ -intercept for the graph of the function.  

$$f(x) = 2 \ln x - 1$$

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12. Use properties of logarithms to expand the expression.

$$\ln \left[ \frac{2x-1}{\sqrt{3x+1}(x^3-7)^9} \right]$$

13. Solve for  $x$ .

$$\ln x - \ln(x-2) = 1$$

14. Solve for zeros.

a.  $f(x) = (x-1)^2(x) - (x-1)$

b.  $g(x) = 3(x^2+4)(x^2+1) + 6(x^2+4)^2$

c.  $h(x) = \sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}$

d.  $k(x) = (x-3)^3(x+2) - 2(x-3)^2(x+2)^2$

e.  $j(x) = (2x+1)^{3/2}(x^{1/2}) + (2x+1)^{5/2}(x^{-1/2})$

15. Reduce each expression to lowest terms.

a.  $\frac{3x+9}{6x}$

b.  $\frac{x^2}{x^{1/2}}$

c.  $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

d.  $\frac{x^{1/2}-x^{1/3}}{x^{1/6}}$

e.  $\frac{\sqrt{x-1}+(x-1)^{3/2}}{\sqrt{x-1}}$

f.  $\frac{1-(\sin x + \cos x)^2}{2 \sin x}$

16. Simplify

a.  $\frac{(x-1)(x+3)-(x+1)^2}{x+1}$

c.  $\frac{x^2-5x+6}{x^2-4x+4}$

b.  $\frac{\sqrt{x^2+1} - \frac{1}{\sqrt{x^2+1}}}{x^2+1}$

d.  $\frac{1}{x+1} - \frac{1}{x-1} - \frac{2}{x^2-1}$

e.  $\frac{x(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$

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17. Rationalize by multiplying by the conjugate of the denominator

a.  $\frac{1}{1-\cos x}$

b.  $\frac{x}{1-\sqrt{x^2+1}}$

c.  $\frac{2}{x+\sqrt{x^2+1}}$

18. Divide

a.  $\frac{x^3-4x^2+2x+1}{x-1}$

b.  $\frac{2x^3+5x+7}{x+1}$

c.  $\frac{x^4-3x^3+x^2+x+2}{x-2}$

19. Find asymptotes (vertical, horizontal and slant).

a.  $f(x) = \frac{x^2-x-2}{x^2-2x-8}$

b.  $f(x) = \frac{2x^2+2x-3}{x+3}$

c.  $f(x) = 3 - \frac{2}{x-1}$

20. Find and verify the inverse of the following functions:

a.  $y = \sqrt{x-2}$

b.  $y = \ln x$

c.  $y = \cos x$

d.  $h(x) = \frac{x+4}{2x-5}$

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21. Evaluate the sine, cosine and tangent of each angle without using a calculator:

Angle	sin	cos	tan
$\frac{\pi}{4}$			
$\frac{\pi}{6}$			
$\frac{\pi}{2}$			
$\frac{\pi}{3}$			
$\pi$			
0			
$\frac{3\pi}{2}$			

You MUST know exact values for the 1<sup>st</sup> quadrant and quadrant angles WITHOUT drawing a unit circle.

22. Find two solutions of each equation. Give your answers in radians  $0 \leq \theta \leq 2\pi$ .

a.  $\cos \theta = \frac{\sqrt{2}}{2}$

b.  $\cos \theta = -\frac{\sqrt{2}}{2}$

c.  $\sec \theta = 2$

d.  $\sec \theta = -2$

e.  $\tan \theta = 1$

f.  $\tan \theta = -\sqrt{3}$

g.  $\sin \theta = \frac{\sqrt{3}}{2}$

h.  $\sin \theta = -\frac{\sqrt{3}}{2}$

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23. Simplify:

a.  $\frac{\cot \theta}{\csc \theta}$

b.  $\frac{\sec \theta}{\tan \theta}$

c.  $(\sin \theta - \sin^3 \theta) \csc \theta$

24. Find all  $\theta$  in the interval  $[0, 2\pi)$  that satisfy the equation:

a.  $2 \cos \theta \tan \theta + \tan \theta = 0$

b.  $\sin \theta - \sin \theta \cot \theta = 0$

c.  $\cos \theta \tan \theta + \sqrt{3} \cos \theta = 0$

d.  $2 \cot \theta \sin \theta - \cot \theta = 0$

For questions 25 to 30, refer to the graph of  $f(x)$  to the right. Find the value of each function or indicated limit. If a limit does not exist, give a reason.

25.  $f(1)$

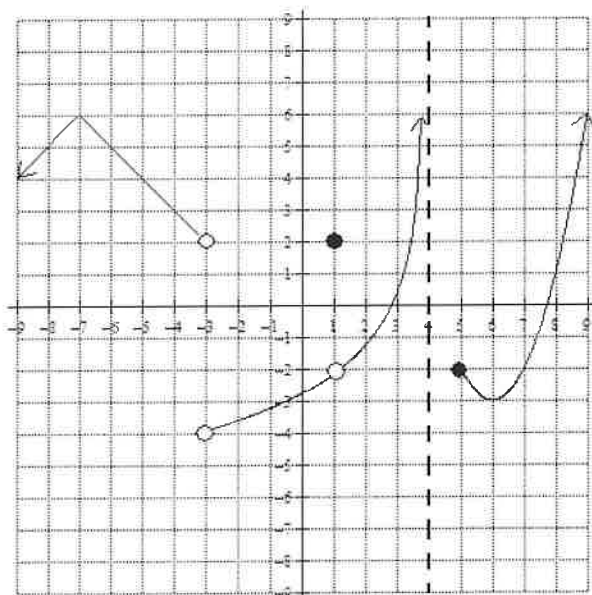
26.  $\lim_{x \rightarrow 1} f(x)$

27.  $\lim_{x \rightarrow -3^+} f(x)$

28.  $\lim_{x \rightarrow -\infty} f(x)$

29.  $\lim_{x \rightarrow 4} f(x)$

30.  $\lim_{x \rightarrow 8} f(x)$



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31. Convert interval notation to inequality notation or vice versa.

a.  $[-2, 3]$

b.  $(-\infty, 0)$

c.  $-4 \leq x \leq 4$

d.  $2 < x < 5$

e.  $-3 < x \leq 5$

f.  $[3, \infty)$

g.  $x \geq 2$

h.  $x < 13$

32. Find all the points of intersection of the graphs of  $x^2 - 2x - y = 6$  and  $x - y = -4$

33. Find the slope and y-intercept for the equation  $4x - 2y + 8 = 0$

34. An open box is to be made from a rectangular piece of material 9 inches by 12 inches by cutting equal squares from each corner and turning up the sides. Let  $x$  be the length of each side of the square cut out of each corner. Write the volume of the box as a function of  $x$ .

35. Find an equation of the line, in point slope form, that passes through the point  $(3, 1)$  and is perpendicular to the line determined by the points  $(8, 9)$  and  $(10, 6)$ .

36. Find  $\frac{g(x + \Delta x) - g(x)}{\Delta x}$  for  $g(x) = x^2 + 3x - 1$

37. Solve for  $x$  using the quadratic formula.  $2x^2 - x = 5$



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38. Factor completely.

a.  $x^2 - 3x + 2$

b.  $x^2 - 9$

c.  $x^2 + 5x - 6$

d.  $x^2 + 5x + 6$

e.  $2x^2 + 5x - 3$

f.  $e^{2x} + 2 + e^{-2x}$

g.  $x^4 - 7x^2 + 12$

h.  $1 - \sin^2 x$

39. Factor by grouping.

a.  $x^3 + 4x^2 - 2x - 8$

b.  $x^3 + 2x^2 + 3x + 6$

c.  $5 \cos^2 x - 5 \sin^2 x + \sin x + \cos x$

d.  $\cos^2 x + 4 \cos x + 4 - \tan^2 x$

40. Factor completely.

a.  $x^3 - 27$

b.  $x^3 - 3x^2 + 3x - 1$

c.  $x^3 + 6x^2 + 12x + 8$

d.  $x^4 - 25$

41. Complete the attached Circuit – Using Tables (pre-calculus)

**Circuit Training - Using Tables (pre-calculus)**

Name \_\_\_\_\_

Directions: The following table shows selected values of three continuous functions  $f$ ,  $g$ , and  $h$ . The function  $h$  is also strictly decreasing. Beginning in cell #1, use only the values in the table to evaluate the expressions or equations for the given  $x$  - value(s). Search for your answer. Call that cell #2 and proceed in this manner until you complete the circuit. For your convenience, the table is on both sides.

<p>Table:</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;"><math>x</math></th> <th style="padding: 5px;"><math>f(x)</math></th> <th style="padding: 5px;"><math>g(x)</math></th> <th style="padding: 5px;"><math>h(x)</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;"><math>\sqrt{2}</math></td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-3</td> <td style="padding: 5px;"><math>\frac{3}{2}</math></td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;"><math>\frac{\pi}{4}</math></td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">4</td> <td style="padding: 5px;">6</td> <td style="padding: 5px;"><math>-\frac{4}{3}</math></td> <td style="padding: 5px;"><math>-\frac{\pi}{2}</math></td> </tr> <tr> <td style="padding: 5px;">5</td> <td style="padding: 5px;">7</td> <td style="padding: 5px;">-3</td> <td style="padding: 5px;">-3</td> </tr> </tbody> </table>	$x$	$f(x)$	$g(x)$	$h(x)$	0	-2	3	4	1	3	$\sqrt{2}$	2	2	0	-3	$\frac{3}{2}$	3	-1	$\frac{\pi}{4}$	0	4	6	$-\frac{4}{3}$	$-\frac{\pi}{2}$	5	7	-3	-3	<p>Answer: <math>-\frac{\pi}{4}</math></p> <p># <u>  1  </u> <math>g(5) \cdot h(2)</math></p>
$x$	$f(x)$	$g(x)$	$h(x)$																										
0	-2	3	4																										
1	3	$\sqrt{2}$	2																										
2	0	-3	$\frac{3}{2}$																										
3	-1	$\frac{\pi}{4}$	0																										
4	6	$-\frac{4}{3}$	$-\frac{\pi}{2}$																										
5	7	-3	-3																										
<p>Answer: <math>\frac{3}{2}</math></p> <p># <u>      </u> <math>3g(1) + 2 \sin(g(3)) + \cos(h(4))</math></p>	<p>Answer: 3</p> <p># <u>      </u> For what value of <math>x</math> does <math>h(x) = g(x)</math> ?</p>																												
<p>Answer: <math>-\frac{9}{2}</math></p> <p># <u>      </u> <math>g(0) - f(1)</math></p>	<p>Answer: 4</p> <p># <u>      </u> <math>\frac{g(3)}{g(4)}</math></p>																												
<p>Answer: <math>-\frac{3\pi}{16}</math></p> <p># <u>      </u> Find <math>g(h^{-1}(0))</math></p>	<p>Answer: <math>\frac{\pi+16}{4}</math></p> <p># <u>      </u> <math>\frac{f(4)}{h(0)}</math></p>																												

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$x$	$f(x)$	$g(x)$	$h(x)$																										
0	-2	3	4																										
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<p>Answer: <math>\frac{\pi}{4}</math></p> <p># _____ If <math>p(x) = \frac{g(x)+5}{f(x)-6}</math>, find <math>p(3)</math>.</p>	<p>Answer: 0</p> <p># _____ <math>f(2) + g(3) + h(0)</math></p>																												
<p>Answer: 5</p> <p># _____ Let <math>h^{-1}(x)</math> be defined as the inverse of <math>h(x)</math>. Find <math>h^{-1}(2)</math>.</p>	<p>Answer: 6</p> <p># _____ Let <math>r(x) = \sqrt{7 - f(x)}</math>. Find <math>r(0)</math>.</p>																												
<p>Answer: <math>4\sqrt{2}</math></p> <p># _____ If <math>p(x) = h(x) - g(x)</math>, find <math>p(1)</math>.</p>	<p>Answer: <math>\frac{\pi+20}{-28}</math></p> <p># _____ Find the average rate of change of <math>h(x)</math> on the closed interval <math>[0, 4]</math>.</p>																												
<p>Answer: <math>-\frac{\pi}{8} - 1</math></p> <p># _____ <math>\text{Arcsin}(f(3)) + \text{Arcsec}(g(1))</math></p>	<p>Answer: 1</p> <p># _____ For what <math>x</math> - value is <math>p(x) = \frac{g(x)+5}{f(x)-6}</math> undefined?</p>																												