West Essex Regional School District

Precalculus CPA

Summer Assignment 2023

Pre-Calculus CPA is a college preparatory course that introduces aspects of higher mathematics. Pre-Calculus consists of those subjects, skills, and insights needed to understand calculus. It includes arithmetic, algebra, coordinate geometry, trigonometry, and, most of all, functions---the general concept as well as specific functions. Students come to this course familiar with basic arithmetic, algebra and geometry. A Pre-Calculus course builds on all of their previous mathematical knowledge and experience. The graphing calculator will be used extensively in this course.

The topics for Precalculus CPA include:

- Understand functions and their graphs
- Use inverse functions to solve equations
- Use basic right triangle trig
- Determine exact unit circle values for given rotations
- Graph trigonometric functions
- Apply and use inverse trig functions
- Recognize and use trig identities
- Discover properties/formulas that apply to trigonometric functions
- Use trigonometric functions to model real-world situations
- Determine area of triangles
- Perform matrix operations
- Use matrices to solve systems in a variety of ways
- Graph logarithmic and exponential functions
- Use logarithmic and exponential functions to model real-world situations
- Graph polynomials using important characteristics
- Find real and imaginary roots of polynomial functions
- Analyze and graph rational functions
- Use graphing calculators to create and model real-life situations

The first major theme of the curriculum is a review of concepts learned in Algebra II and several will be completed over the summer. Upon completing the summer assignment, each student should be able to:

- Identify values within the Real Number System
- Simplify expressions involving exponent
- Solve equations in one variable
- Factor
- Use the distance and midpoint formulas to solve problems
- Graph and write equations for a circle
- Graph and write linear equations

Assignments

- All assignments will be collected on **The First Day Of School**, and counted toward homework grades. No credit will be given for assignments turned in late.
- There will be assessments based on this summer assignment. The dates of the assessments will be announced prior to the quizzes.

Complete all work in the **answer packet showing all work** for each problem and using **Pencil**, credit will not be given otherwise. It is also recommended that students use a three-ring binder for all notes, assignments, and worksheets throughout the school year. Complete all problems in the exercises unless otherwise indicated.
Show all work!

I. Suggested Due Date: 7/31/23

Real Number System---Read page 4 and do page 5 (1-7)
Exponents---Read page 6 and do page 7 (1-20)

II. Suggested Due Date: 8/7/23

Exercises for Solving Equations---Read pages 8-9 and do problem (1-17)
Factoring---Read pages 10-13 and do (1-17) and do page 14 (1-16)

III. Suggested Due Date: 8/14/23

Distance and Midpoint---Read pages 15-16 an do (1-4) and do page 17 (1-11)
Circles---Read pages 18-19 an do (1-6)

IV. Suggested Due Date: 8/21/23

Slope and Rates of Change---Read pages 20-21 and do (1-9)
Read pages 22-24 and do (1-8)
Read page 24-25 and do (1-14)
Quiz---Do page 26 (1-16)

Have a great summer….

Precalculus CPA

Internet Resources for Summer Packet

These internet resources may be helpful to use while completing your summer packet. They will provide another resource of information to review information you have learned in your prior math courses.

www.khanacademy.com

http://mathbits.com/MathBits/TeacherResources/Algebra2/Algebra2.htm

http://www.purplemath.com/

http://www.algebrahelp.com/resources/

www.thatquiz.org

http://www.math.com/homeworkhelp/Algebra.html

http://www.freemathhelp.com/algebra-help.html

http://www.teacherschoice.com.au/mathematics_how-to_library.htm

http://www.algebrahelp.com/

Study Guide

Objective: The Real Number System

Each real number is a member of one or more of the following sets.
The sets of numbers described in the following table should look familiar to you. It is sometimes handy to have names for these sets of numbers, so knowing their names can simplify, for example, describing domains of functions or comprehending theorems such as the rational zeros theorem.

Set	Description
Natural numbers	{1, 2, 3, 4, …. }
Whole numbers	{0, 1, 2, 3, 4, …}
Integers	{ …, -3, -2, -1, 0, 1, 2, 3, …. }
Rational numbers	All numbers that can be written as $\frac{a}{b}$, where a and b are both integers, and b is not equal to 0.
Irrational numbers	Numbers such as $\sqrt{2}, \sqrt[3]{7}, -\pi, e$
Real numbers	The union of the sets of rational numbers and irrational numbers

Things to notice:

*The set of Whole numbers is the same as the set of Natural numbers, except that it includes 0. To help remember this, think “o” as in “whole”.

*The set of Integers is the same as the set of whole numbers and the negatives of the whole numbers.

*We can think of Rational numbers as fractions. To remind us, notice that the word “ratio” is embedded in the word “rational”. A ratio is a fraction.

*The set of Rational numbers includes all decimals that have either a finite number of decimal places or that repeat in the same pattern of digits. For example, 0.3333333… = 1/3 and .245245245… = 245/999.
*The set of Natural numbers is a subset of the set of Whole numbers, which is contained in the set of Integers, which is inside of the set of Rational numbers.

Real Number System Assignment

1. Determine if the following statements are true or false and give a short reason why:

    a. Every integer is a rational number.
    b. Every rational number is an irrational number.
    c. Every natural number is an integer.
    d. Every integer is a natural number.

2. Consider the following set of numbers:

$$\{-\sqrt{49}, -.405, -.\overline{3}, 0, .1, 3, 18, 6\pi, 56, \sqrt{2}\}$$

List all the following:

a) natural numbers:

b) whole numbers:

c) integers:

d) rational numbers:

e) irrational numbers:

f) real numbers:

3. Real numbers are *ordered.* Each real number corresponds to a point on a line. Using 0 as the middle point, draw a number line and label the points 2, π, $-\frac{11}{32}$, 0.

4. The number π is:
a) Real and Rational
b) Irrational and Whole
c) Real and Irrational
d) Rational and Natural

5. It is possible for a number to belong to which two sets?
a) Rational and Irrational
b) Irrational and Integers
c) Rational and Natural
d) Whole and Irrational

6. Classify $\frac{1}{3}$
a) real, rational
b) real, rational, integer
c) real, rational, whole, natural
d) real, rational, whole, natural, integer

7. Which of the choices shows all the rational numbers in this group of numbers?

$$-6, 0, \frac{4}{5}, 1.7, 4.7\overline{63}$$

a) $-6, 0, \frac{4}{5}, 1.7$

b) $-6, 0, \frac{4}{5}, 1.7, 4.7\overline{63}$

c) $-6, \frac{4}{5}, 1.7, 4.7\overline{63}$

d) $0, \frac{4}{5}, 1.7, 4.7\overline{63}$

Study Guide

Objective: Properties of exponents.

Properties of Exponents

Let *a* and *b* be nonzero real numbers. Let *m* and *n* be integers.

Product of Powers $(a)^m(a)^n = a^{m+n}$

Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$

Power of a Power $(a^m)^n = a^{mn}$

Power of a Product $(ab)^n = a^n b^n$

Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

If n is a natural number, then $a^{-n} = \frac{1}{a^n}, a \neq 0$

Zero exponent Property $a^0 = 1, a \neq 0$

Exponent Assignment:

Evaluate the expression. Tell which properties of exponents you used.

1. $2^5 \bullet 2^3$
2. $(-7)^2(-7)$
3. $4^{-6} \bullet 4^{-1}$
4. $(5^{-2})^2$
5. $\frac{4^{-7}}{4^{-3}}$
6. $\frac{8^{-4}}{8^2}$
7. $\left(\frac{2}{3}\right)^3$
8. $\left(\frac{4}{5}\right)^{-3}$

Simplify the expression. Tell which properties of exponents you used.

9. $\frac{x^8}{x^4}$
10. $\frac{y^4}{y^{-7}}$
11. $(3^2s^3)^6$
12. $(4^0w^2)^{-5}$
13. $(y^4z^2)(y^{-3}z^{-5})$
14. $(2m^3n^{-1})(8m^4n^{-2})$
15. $(7c^7d^2)^{-2}$
16. $(5g^4h^{-3})^{-3}$
17. $\frac{x^5y^{-8}}{x^5y^{-6}}$
18. $\frac{16q^0r^{-6}}{4q^{-3}r^{-7}}$
19. $\frac{12a^{-3}b^9}{21a^2b^{-5}}$
20. $\frac{8e^{-4}f^{-2}}{18ef^{-5}}$

Study Guide

Objective: Solve linear equations.

Vocabulary

An **equation** is a statement that two expressions are equal.

A **linear equation** in one variable is an equation that can be written in the form $ax + b = 0$ where "a" and "b" are constants and $a \neq 0$.

A number is a **solution** of an equation if substituting the number for the variable results in a true statement.

Two equations are **equivalent equations** if they have the same solution(s).

EXAMPLE 1

Solve an equation with a variable on one side

Solve 6 x –8 = 10

Solution

$6x - 8 = 10$	Write original equation
$6x = 18$	Add 8 to each side
$x = 3$	Divide each side by 6.

EXAMPLE 2

Solve an equation with a variable on both side

Solve $8z + 7 = -7 - 2z - 3$

Solution

$8z + 7 = -2z - 3$	Write original equation
$10z + 7 = -3$	Add 2z to each side.
$10z = -10$	Subtract 7 from each side
$z = -1$	Divide each side by 10.

Exercises for Examples 1 and 2

Solve the equation. Check your solution.

1. $14x = 7$
2. $3n + 2 = 14$
3. $-6t - 5 = 13$
4. $11q - 4 = 6q - 9$
5. $5a - 1 = 2a + 11$
6. $-2m + 3 = 7m - 6$
7. $11p - 9 + 8p - 7 + 14p = 12p + 9p + 4$

EXAMPLE 3

Solve an equation using the distributive property

Solve 2(3x + 1) = –3(x –2).

Solution

$2(3x + 1) = -3(x - 2)$	Write original equation
$6x + 2 = -3x + 6$	Distributive property
$9x + 2 = 6$	Add 3*x* to each side
$9x = 4$	Subtract 2 from each side
$x = \frac{4}{9}$	Divide each side by 9

Exercises for Example 3

8. Solve $4(2x - 1) = 3(x + 2)$.

9. Solve $5(x + 3) = -(x - 3)$.

10. Solve $2y + 3(y - 4) = 2(y - 3)$.

11. Solve $-9m - (4 + 3m) = -(2m - 1) - 5$.

EXAMPLE 4

Solve multiple step problem

Solve $\frac{7}{8}p - 4 = 10$ **** CLEAR FRACTIONS FIRST****

Solution

$\frac{7}{8}p - 4 = 10$	Write original equation
$8\left(\frac{7}{8}p - 4 = 10\right)$	Multiple every term by the LCD
$7p - 32 = 80$	Add 32 to each side
$7p = 112$	Divide each side by 7
$p = 16$	

Exercises for Example 4

12. Solve $\frac{x}{-5} + 3 = -13$

13. Solve $-4 = \frac{7x - (-1)}{-8}$

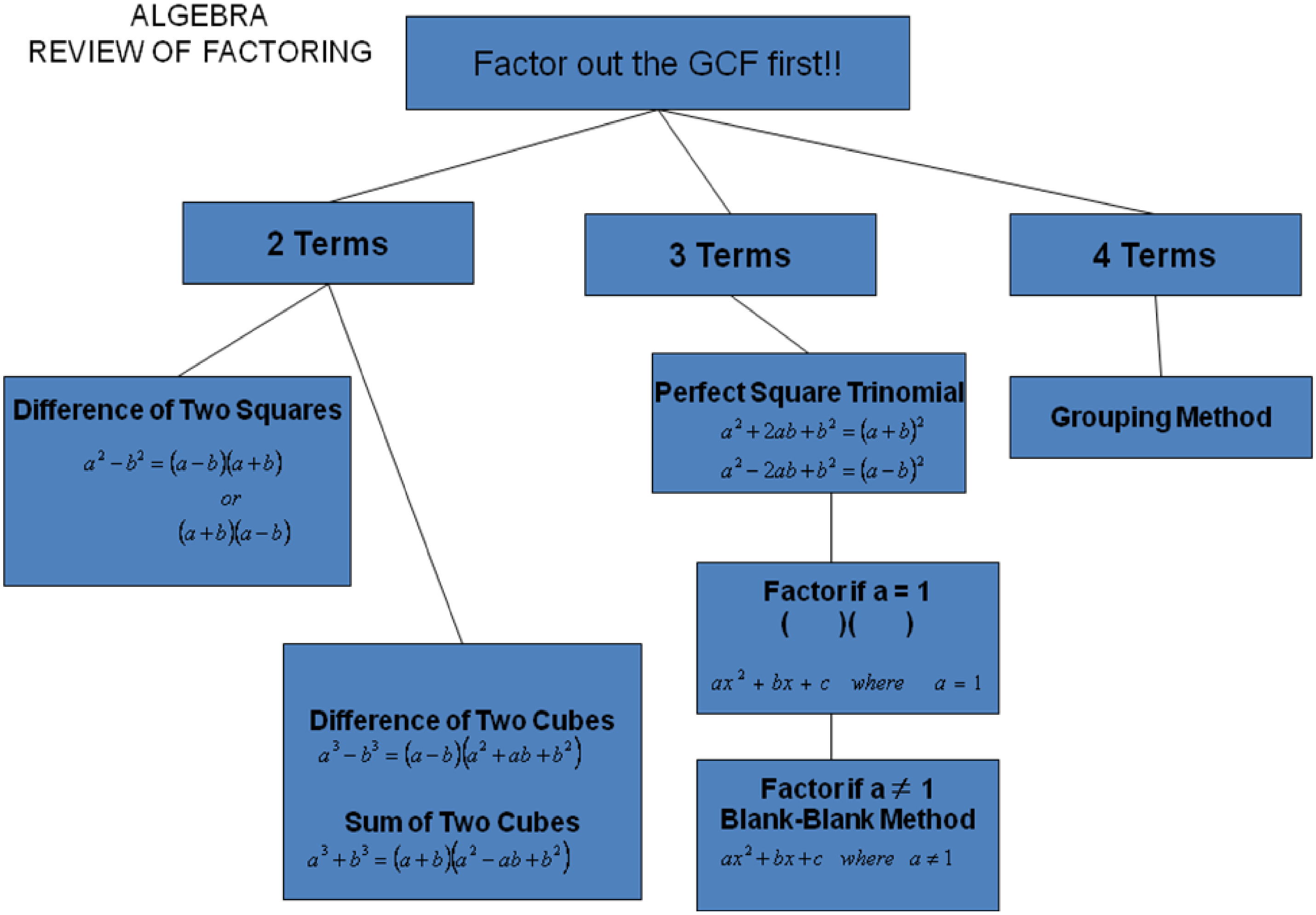
14. Solve $\frac{3k}{5} - \frac{2k}{3} = 1$

15. Solve $\frac{m-2}{3} + \frac{m}{4} = \frac{1}{2}$

16. Solve .05k + .12(k + 5000) = 940

17. Solve .02(50) + .08r = .04(50 + r)

ALGEBRA
REVIEW OF FACTORING



Study Guide
Objective: Factor Expressions

Factoring is the reverse of multiplying.

Multiplying → $2x(x+3) = 2x^2 + 6x$ ← Factoring

Multiplying → $(x + 5)(x + 2) = x^2 + 7x + 10$ ← Factoring

***GCF**
To factor an expression containing two or more terms, factor out the GCF.
1.) Find GCF
2.) Express polynomial as product of the GCF and its remaining factors

Ex 1 Factor each expression

1. $20x^4 + 36x^2$

2. $3x(4x + 5) - 5(4x + 5)$

3. $5x^3 - 10x^2 + 25x + 50$

***Grouping Method**
Use the grouping method to factor polynomials with four terms
1.) Group terms that have common factors
(do not group 1st term w/last term)
2.) Factor the GCF from each binomial
3.) Binomial factors must match
4.) Express answer as a product of the two binomials

Ex 2 Factor each quadratic expression

4. $15x^2 - 3x + 10x - 2$

5. $8x^2 - 5x - 24x + 15$

***Difference of Two Squares**

$$a^2 - b^2 = (a+b)(a-b) = (a-b)(a+b)$$

Ex 3 Factor each quadratic expression

6. $4x^2 - 25$

7. $36x^2 - 81$

***Sum or Difference of Cubes**

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Ex 4 Factor each binomial

8. $y^3 - 64$

9. $8x^3 + 125$

***Factoring Perfect-Square Trinomials**

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^2 - 2ab + b^2 = (a-b)^2$$

1.) Is the first term a perfect square?
2.) Is the last term a perfect square?
3.) Is the middle term equal to twice the product of the square root of the first term and the square root of the last term?

Ex 5 Factor each quadratic expression

10. $9x^2 - 42x + 49$

11. $16x^2 + 72x + 81$

***Factoring ax² + bx + c where a = 1 using integers**.

$x^2 + bx + c = (x + r)(x + s)$

Quadratic trinomials of the form $ax^2 + bx + c$ can be factored using integers if and only if there are two integers whose product is *ac* and whose sum is *b*.

Find two <u>integers</u> *r* and *s* whose product equals the last term *(c)* and whose sum equals the coefficient of the middle term *(b).*

Ex 6 Factor each quadratic expression

12. $x^2 + 5x + 6 =$

13. $x^2 - 7x + 10 =$

14. $x^2 + 3x - 10 =$

15. $x^2 - 7x - 30 =$

***Factoring $ax^2 + bx + c$ where $a \neq 1$ using integers**.

1.) Find two integers whose product is equal to the product of the first and last terms *(ac)*, and whose sum equals the coefficient of the middle term *(b)*.
2.) Rewrite the middle term using the two integers.
3.) Use the grouping method and express your answer as a product of two binomials.

Ex 7 Factor each quadratic expression

16. $8x^2 - 14x + 3$

17. $3x^2 + 11x - 20$

Factor Completely Assignment

1. $x^2 + 12x + 36$

2. $x^2 - 144$

3. $b^2 - b + 2b - 2$

4. $4t^2 + 4t - 15$

5. $-9x^2 + 27x$

6. $a^3 - 27$

7. $16x^2 - 81$

8. $9x^2 + 24x + 16$

9. $8x^3 + 729y^3$

10. $4x^2 + 25$

11. $6x^2 - 7xy + 2y^2$

12. $a^3 + 9a^2 + 18a$

13. $5x^2 - 20x + 10$

14. $2xy - x^2y + 6 - 3x$

15. $3a^2 - 10a + 8$

16. $x^2 + 2x - 63$

Study Guide

Objective: Find the length and midpoint of a line segment

Vocabulary

The **distance formula** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ gives the distance d between the points (x_1, y_1) and (x_2, y_2).

The **midpoint formula** $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ gives the midpoint M of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$.

EXAMPLE 1

Find the distance between two points

Find the distance between (–2, –4) and (3, 4).

Solution

Let $(x_1, y_1) = (-2, -4)$ and $(x_2, y_2) = (3, 4)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-(-2))^2 + (4-(-4))^2} = \sqrt{25+64} = \sqrt{89}$$

EXAMPLE 2

Classify a triangle using the distance formula

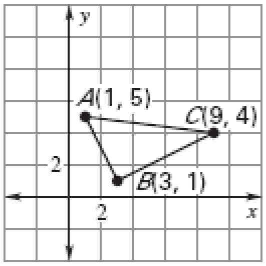
Classify Δ*ABC* as *scalene, isosceles,* or *equilateral.*

$$AB = \sqrt{(3-1)^2 + (1-5)^2} = \sqrt{20} = 2\sqrt{5}$$

$$BC = \sqrt{(9-3)^2 + (4-1)^2} = \sqrt{45} = 3\sqrt{5}$$

$$AC = \sqrt{(9-1)^2 + (4-5)^2} = \sqrt{65}$$

Because $AB \neq BC \neq AC$, $\Delta\ ABC$ is scalene.



Exercises for Examples 1 and 2

1. Find the distance between (1, 5) and (2, −4).
2. The vertices of a triangle are $X(-3, -1)$, $Y(0, 3)$, and $Z(3, -1)$. Classify ΔXYZ as *scalene, isosceles,* or *equilateral.*

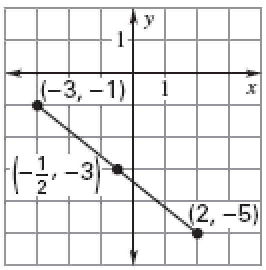
EXAMPLE 3

Find the midpoint of a line segment

Find the midpoint of the line segment joining (–3, –1) and (2, –5).

Let $(x_1, y_1) = (-3, -1)$ and $(x_2, y_2) = (2, -5)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3+2}{2}, \frac{-1+(-5)}{2}\right) = \left(-\frac{1}{2}, -3\right)$$



EXAMPLE 4

Using the midpoint formula to solve

For $\overline{PQ}$, the coordinates of P and M, the midpoint of $\overline{PQ}$, are given. Find the coordinates of Q.

Let P(-2, 3) and M(5, 1).

$$\frac{-2+x_2}{2} = 5;\ x_2 = 12$$

$$\frac{3+y_2}{2} = 1;\ y_2 = -1$$

Therefore Q = (12, -1)

Exercises for Example 3 and 4

For the line segment joining the two given points, find the midpoint

3. (0, 0), (6, −4)

For $\overline{PQ}$, the coordinates of P and M, the midpoint of $\overline{PQ}$, are given. Find the coordinates of Q.

4. P(-1, 4) and M (2, -3)

Distance/Midpoint Assignment

Find the distance between the two points. Then find the midpoint of the line segment joining the two points.

1. $(-9, 7), (3, -4)$
2. $(-2.8, 6.1), (-1.2, 2.5)$
3. $(-7.9, 0.1), (6.8, -9.2)$
4. $\left(4, \frac{5}{2}\right), \left(\frac{3}{2}, 2\right)$

The vertices of a triangle are given. Classify the triangle as *scalene, isosceles,* or *equilateral.*

5. $(2, 5), (-2, 8), (-4, -1)$
6. $(-7, 2), (6, -3), (4, 2)$
7. $(1, 3), (8, 7), (5, 10)$

Use the given distance *d* between the two points to find the value of *x* or *y*.

8. $(-12, 7), (x, -10); d = \sqrt{545}$
9. $(2.3, y), (6.9, 8.5); d = \sqrt{74.45}$

For $\overline{PQ}$, the coordinates of P and M, the midpoint of $\overline{PQ}$, are given. Find the coordinates of Q.

10. Let P(-3, 4) and M(1, 1).
11. Let P(2, 3) and M(-3, -2).

Objective: Graph and write equations of circles.

Vocabulary

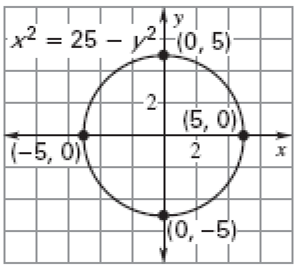
A **circle** is the set of all points (x, y) in a plane that are equidistant from a fixed point, called the **center** of the circle. The distance r between the center and any point (x, y) on the circle is the **radius**.

EXAMPLE 1

Graph an equation of a circle

Graph $x^2 = 25 - y^2$. Identify the radius of the circle.

STEP 1 Rewrite the equation $x^2 = 25 - y^2$ in standard form as $x^2 + y^2 = 25$.



STEP 2 Identify the center and radius. From the equation, the graph is a circle centered at the origin with radius $r = \sqrt{25} = 5$.

STEP 3 Draw the circle. First plot convenient points that are 5 units from the origin, such as (0, 5), (0, −5), (5, 0), and (−5, 0). Draw the circle that passes through the points.

EXAMPLE 2

Write an equation of a circle

The point (6, 2) lies on a circle whose center is the origin. Write the standard form of the equation of the circle.

Because the point (6, 2) lies on the circle, the circle's radius must be the distance between the center (0, 0) and the point (6, 2). Use the distance formula.

$$r = \sqrt{(6-0)^2 + (2-0)^2} = \sqrt{36+4} = \sqrt{40}$$ The radius is $\sqrt{40}$

Use the Standard form with $r = \sqrt{40}$ to write an equation of the circle

$x^2 + y^2 = r^2$ Standard form

$x^2 + y^2 = \left(\sqrt{40}\right)^2$ Substitute $\sqrt{40}$ for r.

$x^2 + y^2 = 40$ Simplify

Exercises for Examples 1 and 2

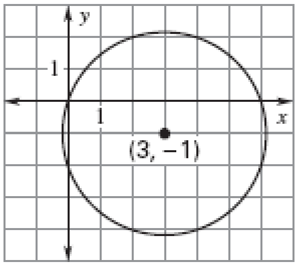
Graph the equation. Identify the center and the radius of the circle.

1. $x^2 + y^2 = 16$
2. $x^2 = 81 - y^2$
3. $y^2 - 9 = -x^2$
4. Write the standard form of the equation of the circle that passes through (3, 2) and whose center is the origin.

Example 3

Write an equation of a translated circle

Write an equation of the circle with center at (3, −1) and radius $r = \sqrt{10}$



STEP 1 Determine the form of the equation. The equation has the form $(x - h)^2 + (y - k)^2 = r^2$.

STEP 2 Identify h and k. The center is at (3, −1), so $h = 3$ and $k = -1$.

STEP 3 Find r^2. Because $r = \sqrt{10}$, $r^2 = 10$.

The standard form of the equation is $(x - 3)^2 + (y + 1)^2 = 10$.

Exercises for Examples 3

Graph the equation. Identify the important characteristics of the graph.

5. $(x - 5)^2 + (y + 1)^2 = 9$
6. Write an equation of a circle with center (–4, 2) and radius $r = 3$.

Study Guide

Objective: Find slopes of lines and rates of change.

Vocabulary

The slope *m* of a non-vertical line through points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the rise) to horizontal change (the run):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two lines are **parallel** if and only if they have the same slope, $m_1 = m_2$

Two lines are **perpendicular** if and only if their slopes are negative reciprocals of each other: or $m_1 \bullet m_2 = -1$.

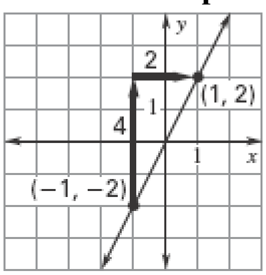
$$m_1 = -\frac{1}{m_2}$$

Slope can be used to represent an average **rate of change**, or how much one quantity changes, on average, relative to the change in another quantity.

EXAMPLE 1

Find the slope of a line

What is the slope of the line passing through the points (–1, –2) and



Let $(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (1, 2)$.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-2)}{1-(-1)} = \frac{4}{2} = 2$$

The slope of the line is 2.

Exercises for Example 1

1. What is the slope of the line passing through the points (0, 0) and (1, 3)?
2. What is the slope of the line passing through the points (2, 1) and (3, - 1)?

EXAMPLE 2

Classify lines using slope

Without graphing, tell whether the line through the given points *rises, falls, is horizontal, or is vertical*.

a. (4, 2), (3, 2)
b. (1,4), (5, 7)
c. (4, 1), (4,-2)
d. (-1,2), (1,0)

Solution

a. Because $m = 0$, the line is horizontal.
b. Because $m > 0$, the line rises.
c. Because m is undefined, the line is vertical.
d. Because $m < 0$, the line falls.

Exercises for Example 2

Without graphing, tell whether the line through the given points rises, falls, is horizontal, or is vertical.

3. (–4, 2), (1,0)
4. (1,6), (1,0)
5. (0,–3), (4,–3)
6. (–1,–1), (2,0)

EXAMPLE 3

Classify parallel and perpendicular lines

Tell whether the lines are parallel, perpendicular, or neither.
Line 1: through (0, 2) and (1, 3)
Line 2: through (1, 0) and (2, –1)

The slope of Line 1 is 1. The slope of Line 2 is –1. Because $m_1 \bullet m_2$ is –1, m_1 and m_2 are negative reciprocals of each other and the lines are perpendicular.

Exercises for Example 3

Tell whether the lines are *parallel, perpendicular, or neither.*

7. Line 1: through (–1, –1) and (1, 3)
Line 2: through (–2, –2) and (1, 4)
8. Line 1: through (1, 5) and (0, 3)
Line 2: through (2, –3) and (0, 1)

EXAMPLE 4

Solve a multi-step problem

Use the table, which shows the growth of human hair over 4 months, to find the average rate of change in the length of human hair over time. Then predict the length of human hair at 9 months.

Month	1	2	3	4
Length of hair (in inches)	6	6.4	7	7.5

STEP 1

Find the average rate of change.

Average rate of change = $\dfrac{\text{change in length}}{\text{change in time}} = \dfrac{7.5 - 6}{4 - 1} = \dfrac{1.5}{3}$ inch per month

STEP 2

Predict the length of human hair at 9 months. Find the increase in the length of hair from 4 months to 9 months.

Increase in length = (5 months)(0.5 inch per month) = 2.5 inches

Now add this amount to the length of the hair at 4 months. At 9 months, the length of human hair will be about 7.5 + 2.5 = 10 inches.

Exercise for Example 4

9. Use the average rate of change from Example 4 to predict the length of human hair at 11 months.

Objective: Graph linear equations in slope-intercept or standard form.

Vocabulary

The **parent function** is the most basic function in a family.

A ***y*-intercept** of a graph is the *y*-coordinate of a point where the graph intersects the *y*-axis.

The equation $y = mx + b$ is said to be in **slope-intercept form**.

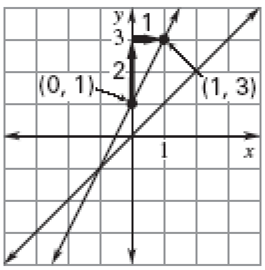
The **standard form of a linear equation** is $Ax + By = C$ where *A* and *B* are not both zero. In addition, **A cannot be negative, and A, B, and C cannot be fractions!**

An ***x*-intercept** of a graph is the x-coordinate of a point where a graph intersects the x-axis.

EXAMPLE 1

Graph an equation in slope-intercept form

Graph the equation $y= 2x+ 1$. Compare the graph with the graph of $y = x$.



STEP 1 The equation is already in slope-intercept form.

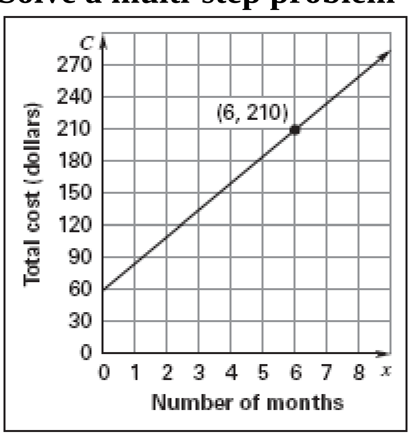
STEP 2 Identify the *y*-intercept. The *y*-intercept is 1, so plot the point (0, 1).

STEP 3 Identify the slope. The slope is 2. Plot a second point by starting at (0, 1) and then moving up 2 units and right 1 unit. The second point is (1, 3).

STEP 4 Draw a line through the two points. The graph of $y = 2x + 1$ has a *y*-intercept of 1, but the graph of $y = x$ has a *y*-intercept of 0. And, the graph of $y = 2x + 1$ has a slope of 2, but the graph of $y = x$ has a slope of 1.

EXAMPLE 2

Solve a multi-step problem



A tennis club charges an initiation fee of $60 and a monthly fee of $25. Graph the equation $C= 25x+ 60$ that represents the cost *C* of membership. Describe what the slope and *y*-intercept represent. Estimate the total cost after 6 months.

The slope represents the monthly fee and the *y*-intercept represents the initiation fee. From the graph you can see that the total cost after 6 months is $210.

Exercises for Examples 1 and 2

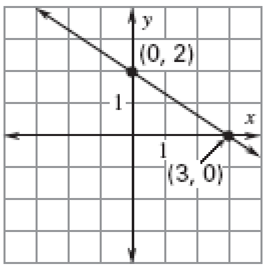
Graph the equation. Compare the graph with the graph of $y = x$.

1. $y = x + 1$
2. $y = 5x - 3$
3. $y = -3x + 2$
4. Rework Example 2 to find the total cost of your membership after 9 months if the initiation fee is $50 and the monthly fee is $20.

EXAMPLE 3

Graph an equation in standard form

Graph $2x + 3y = 6$.



STEP 1 The equation is already in standard form.

STEP 2 Identify the *x*-intercept.

$2x + 3(0) = 6$ Let $y = 0$.

$x = 3$ Solve for *x*.

The *x*-intercept is 3. So, plot the point (3, 0).

STEP 3 Identify the y-intercept.

$2(0) + 3.y = 6$ Let $x = 0$.

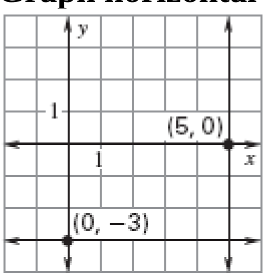
$y = 2$ Solve for *y*.

The *y*-intercept is 2. So, plot the point (0, 2).

STEP 4 Draw a line through the two points.

EXAMPLE 4

Graph horizontal and vertical lines



Graph the equation in a coordinate plane.

a. $y = -3$
b. $x = 5$

Solution

a. The graph of $y = -3$ is the horizontal line that passes through the point (0. –3).
b. The graph of $x = 5$ is the vertical line that passes through the point (5, 0)

Exercises for Examples 3 and 4

Graph the equation.

5. $x + 2y = 4$
6. $5x + 3y = 15$

7. $x = -1$
8. $y = 4$

Objective: Write linear equations.

Vocabulary

The **point-slope form** of the equation of a line is given by $y - y_1 = m(x-- x_1)$, where m is the slope and (x_1,y_1) is a point on the line.

EXAMPLE 1

Write an equation given the slope and a point

a. Write an equation with a slope of –2 and ay-intercept of 3.
b. Write an equation with a slope of 4 that passes through the point (– 1, 7).

Solution

a. $y = mx + b$	Use slope-intercept form.
$y = -2x + 3$	Substitute –2 for m and 3 for b.
b. $y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 7 = 4(x + 1)$	Substitute for m, x_1 and y_1
$y - 7 = 4x + 4$	Distributive property
$y = 4x + 11$	Write in slope-intercept form.

EXAMPLE 2

Write equations of parallel or perpendicular lines

Write an equation of the line that passes through (0, 1) and is (a) parallel to, and (b) perpendicular to, the line y= 2x+ 7.

Solution

a. The given line has a slope of $m_l = 2$. A line parallel to the given line has a slope of $m_2 = 2$. Use the point-slope form with $(x_1,y_1) = (0, 1)$ and $m_2 = 2$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 1 = 2(x - 0)$	Substitute for m, x_1, and y_1.
$y - 1 = 2x - 2(0)$	Distributive property
$y = 2x + 1$	Write in slope-intercept form.

b. A line perpendicular to a line with slope $m_l = 2$ must have a slope of

$m_2 = -\frac{1}{m_1} = -\frac{1}{2}$ Use the point-slope form with

$(x_1,y_1) = (0, 1)$.

$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 1 = -\frac{1}{2}(x - 0)$	Substitute for x_1 and y_1
$y - 1 = -\frac{1}{2}x - \left(-\frac{1}{2}\right)(0)$	Distributive property
$y = -\frac{1}{2}x + 1$	Write in slope-intercept form.

Exercises for Examples 1 and 2

Write an equation of the line with the given conditions.

1. With a slope of 4 and a y-intercept of –1
2. With a slope of –5 that passes through the point (3, –2)
3. That passes through (2, 3) and parallel to $y = -x + 3$
4. That passes through (0, 1) and perpendicular to $y = 2x + 1$

EXAMPLE 3

Write an equation given two points

Write an equation of the line that passes through (2, 8) and (4, 14).

Find the slope of the line through $(x_1, y_1) = (2, 8)$ and $(x_2, y_2) = (4, 14)$.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14-8}{4-2} = \frac{6}{2} = 3$$

Use the point-slope form with the point (2, 8) to write an equation.

$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 8 = 3(x - 2)$	Substitute for m, x_1, and y_1
$y - 8 = 3x - 6$	Distributive property
$y = 3x + 2$	Write in slope-intercept form.

Exercises for Example 3

Write an equation of the line that passes through the given points.

5. (1,–1), (4, 2)
6. (–2, 4), (3,–1)
7. (–3,–1), (0, 1)

8. Find the x- and y-intercepts for the equation 3x - 2y = 8.

9. Find the slope of the line passing through the points (6,4) and (-1,2).

10. Write the equation of the line passing through the point (-1,3) and has the slope of $-\frac{2}{5}$.

11. Write the equation of the line passing through the point (0,4), perpendicular to the line $y = -\frac{5}{4}x$.

Extension

Write an equation for the perpendicular bisector of the line segment joining the two points.

12. (–5, 4), (4, –5)
13. (2, 7), (2, –5)
14. (–9, 4), (1, 1)

Quiz

1) Put an "x" in each box for which the number on the left of the chart belongs to the set of numbers across the top.

	Integer	Rational	Irrational	Real	Natural	Whole
a. 5						
b. $.\overline{25}$						
c. -7						
d. $\sqrt{3}$						
e. $\sqrt{16}$						
f. 0						
g. π						
h. 1.765						
i. $-\frac{17}{5}$						
j. $-\sqrt{6}$						

Simplify each expression.

2) d^2d^4

3) $(9y^{-4})^2$

4) $\frac{(xy^3)^2}{xy^{-1}}$

Solve each equation.

5) $6x - 2 = 5x - 7 - 3x$

6) $3(8x - 5) + 3 = 22x + 2(x - 6)$

7) $\frac{6x+7}{4} + \frac{3x-5}{7} = \frac{5x+78}{28}$

Factor Completely.

8) $2x^3 + 128$

9) $x^2 - 49$

10) $x^2 - 3x - 28$

11) $3x^2 - 8x + 5$

12) Find a) the distance and b) the midpoint between the points (4, 1) and (1, 4).

13) Identify the center and radius of the circle given the equation $(y - 1)^2 + (x - 5)^2 = 25$

14) Write the equation of the circle with center (3, -2) and radius of $\sqrt{5}$.

15) Identify the slope and y-intercept of the equation $y = \frac{2}{5}x + 4$.

16) Write the equation of the line that goes through (1, -2) and (-3, -5).